Laser Gyroscope
Experimental Instructions

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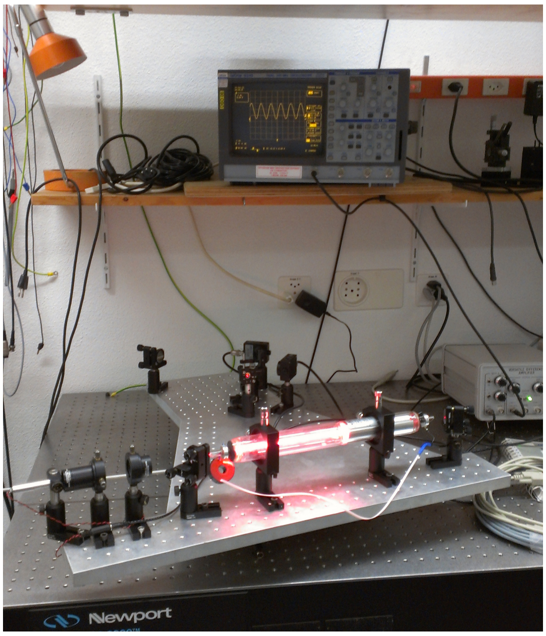

Figure 1: Laser gyroscope in use.

Abstract

This experiment is designed to demonstrate an important application of lasers. The correct handling of lasers and optical components and in particular their alignment is learned.

Contents

1 Introduction

Laser gyrosocopes are used in some very important applications. They are applied whenever a positioning in space independent of gravitational fields and centrifugal forces is needed, or where the timescale of changes does not allow for the use of GPS positioning. Examples are scientific and civil satellites, commercial and military aeronautics. In geodesy, laser gyroscopes are used to continually measure the rotation of the earth and monitor any changes. Depending on the purpose, they differ in size and precision. Modern laser gyroscopes are capable of resolving rotations down to $\pm$ 0.01°/h [6] (this corresponds to one full revolution in four years), which makes them much more accurate than mechanical gyroscopes.

2 Theoretical Background

2.1 Principle of a Laser

A LASER (**L**ight **A**mplifiction by **S**timulated **E**mission of **R**adiation) generates a monochromatic beam of light of high intensity a large coherence length. A laser consists of a resonator, a pump (energy) source and an active medium that determines the wavelength of the light.

Light amplification happens inside the active medium. Hereby, the principle of stimulated emission is utilised: similar to spontaneous emission, where an atom randomly relaxes from an excited to a lower energy level under emission of a photon, stimulated emission has the process triggered by a passing photon (see fig. 2). In contrast to spontaneous emission, where the photon is emitted isotropically and with random phase, the photon emitted through stimulated emission exhibits the same direction and phase as the incoming (stimulating) photon.

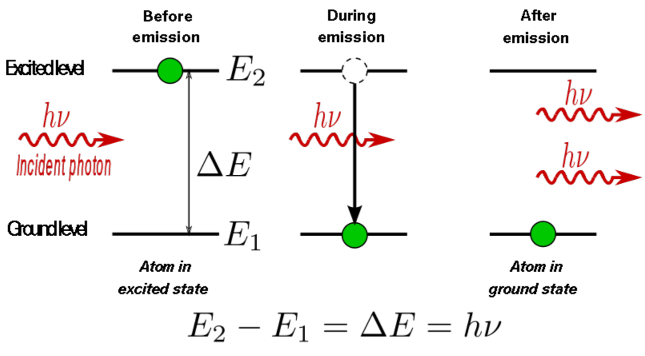

Figure 2: Illustration of the process of stimulated emission. In contrast to spontaneous emission, stimulated emission produces photons with the same direction and phase.[2]

[2]Source: http://en.wikipedia.org/wiki/File:Stimulated_Emission.svg, 11.07.2012

Inside the resonator, certain conditions must be met to allow for amplification of the light. Firstly, the photons must be kept within the resonator using mirrors, so that they must repeatedly transit the active medium. Thus, a standing wave can form between the mirrors. This is what ensures that emitted photons are always in phase and interfere constructively rather than destructively with the existing photons. Wavelength and resonator length are not independent of each other. To establish a standing wave, the resonator length must be an integer multiple of the light wavelength. The mode number m describes the number of oscillations inside the resonator. Figure 3 shows two linear resonators with mode numbers m = 5 (resonator A) and m = 8 (resonator B).

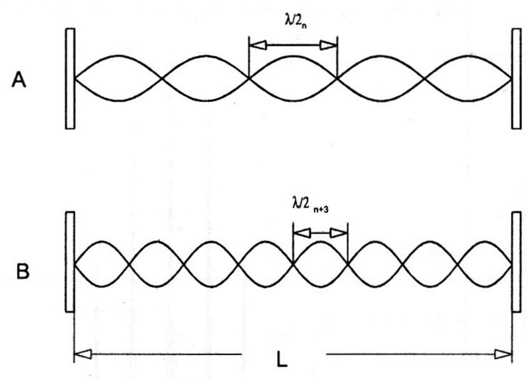

Figure 3: Standing waves with different mode numbers inside a linear resonator. [2]

Secondly, while a photon can stimulate an excited atom to emit another photon, the probability of it being absorbed by an atom in a lower energy state is exactly the same. To gain a net increase in photons (and thus amplification of the light), the number of atoms in the excited state (from which they can be stimulated to emit a photon) must be larger than the number of atoms in the lower energy state (from which they can absorb a photon and thus climb to the excited state). Only then will the process of stimulated emission outweigh that of spontaneous absorption. This state is called population inversion and must be sustained by constantly adding energy to the gain medium (called pumping). This can be achieved (depending on the gain medium) optically (through a second laser), chemically or electrically. If the gain medium is a gas (such as in a HeNe laser), the pump energy is usually fed into the medium via an electric current. A large voltage is required to ionize the gas atoms, after which a current can flow. The collisions between flowing electrons and gas atoms are exciting the atoms and creating the population inversion. Figure 4 schematically shows the excitation of a HeNe gain medium with all involved energy levels.

Finally, to obtain a beam outside the resonator, one of the mirrors is designed with a low transmission rate (a few percent).

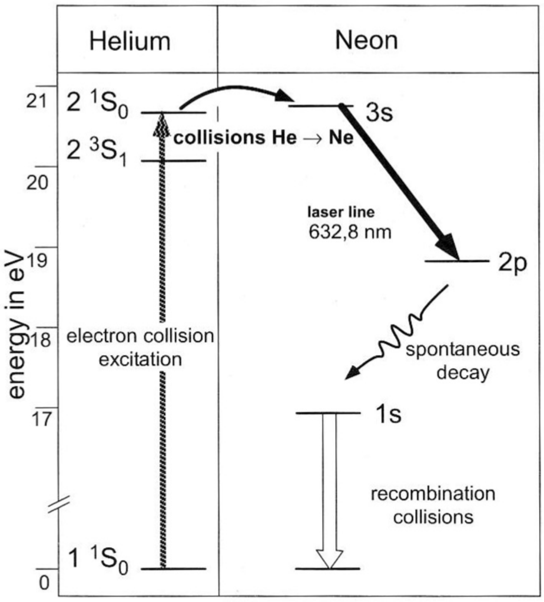

Figure 4: Illustration of energy levels and transitions in a HeNe laser gain medium. [3, S. 193]

2.2 Principle of a Ring Laser

A ring laser is a special type of resonator, where the photons leaving the gain medium are guided around and back into the gain medium from the other side. This can be done with mirrors or with optical fibres. This setup has the consequence that there is a beam formed in clockwise (cw) and another in counter-clockwise (ccw) direction.

2.3 Stability of a Ring Laser

The length of the resonator and the radii of curvature of the mirrors need to be adjusted according to each other, or else the resonator will not lase. The beam in the resonator has a non-zero divergence, which means that the beam diameter will increase with every round trip through the resonator (assuming plane mirrors are used). This means that eventually, the beam diameter will be too large to fit either the mirrors or the active medium (or opening thereof), resulting in high losses and no lasing. To counteract this, it is necessary to keep focusing the beam by changing one or more plane mirrors to a curved mirror. Curved mirrors will focus the beam to a more narrow diameter and so will, if curvature was appropriately chosen, cancel out the divergence. The stability criterion explains the relationship between resonator length (L) and mirror curvature (radius of curvature R_c) to attain stability. If resonator length and mirror curvature do not obey this criterion, the laser can not operate.

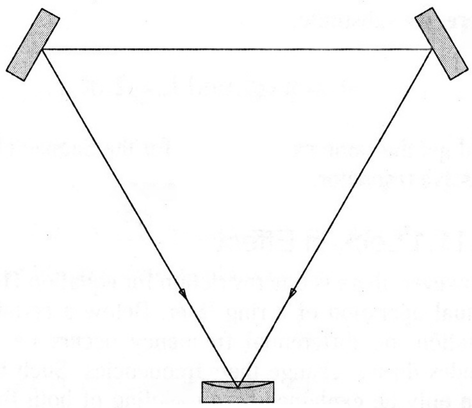

Figure 5: Geometry of the resonator used in this experiment. [2]

Figure 5 shows the geometry of the triangular resonator made up of two plane and one curved mirror.

For this setup the following stability criterion holds:

$$-1 \leq g \leq 1 \tag{2.1}$$

with $g = 1 - \frac{L}{R_c}$.

The quantitative description of the light-amplifying capability of a laser is called gain. Gain increases, everything else being equal, with the length of the active medium and decreases with increasing resonator length (if the gain medium remains the same length). Gain also depends on the quality (and cleanness) of the mirrors and the precision of their alignment. A laser with higher gain will produce a beam with higher intensity and at the same time be easier to align. Any deviation of the mirror positions from the optimum will lower the gain of the system, until it falls below the lasing threshold, where operation is no longer possible.

2.4 Sagnac Effect

Let us assume an observer sits in a rotating system and sends the light signals in opposite direction on a circular path. The two signals will, after a full revolution, arrive at two separate times in the starting point (see figure 6). This leads to a phase shift for continuous signals. This effect is called Sagnac effect. The phase shift can be measured, and from it the angular velocity (rotational velocity) Ω can be calculated.

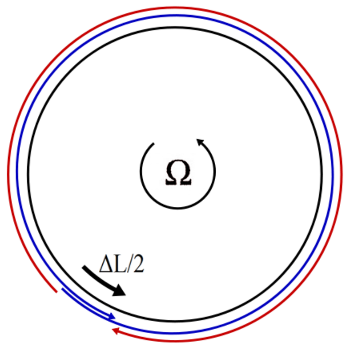

Figure 6: Schematic representation of the Sagnac effect.[3]

The problem is that a rotation of this setup ("passive Sagnac interferometer") leads to a phase shift much smaller than a wavelength, which is hardly measurable. This kind of laser gyroscope is thus exclusively built as an optical fibre variant with many hundred windings and a length of up to a few kilometers. Thus, the phase shift can be strongly increased and easily measured to high precision.
In our experiment however, we use a ring resonator ("active Sagnac interferometer"), and a rotation will not result in a phase shift. Instead, the beams will shift in frequency. For the ring laser to lase, the resonator length must be an integer multiple of the wavelength (resonance condition, as explained in section 2.1). Upon rotation, the mode number in the ring does not change, but the frequencies of the two beams are shifted to still meet the resonance condition (compare fig. 7).
The Sagnac effect in a ring laser thus describes an antisymmetric frequency shift of the two beams: for example a rotation in cw direction will result in a frequency upshift of the ccw beam and a frequency downshift of the cw beam. This shift can be measured by letting the two beams interfere with each other, resulting in a beat frequency $\Delta\nu$, often called Sagnac frequency because of its origin.
In order to calculate the rotational velocity Ω from the measured beat frequency $\Delta\nu$, the relation between the two must be established. As the frame of reference is rotating (and thus accelerated), only general relativity can fully describe the process. The classical derivation, which is used in this instruction, permits an approximation to first order[4].

[3]Source: http://en.wikipedia.org/wiki/File:Sagnac_shift.svg, 11.07.2012
[4]The following derivation is based on [1, S. 135ff.]

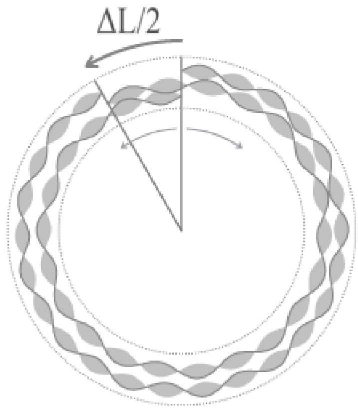

Figure 7: Frequency shifts in a rotating ring resonator.[6] The light in both the cw and ccw direction goes through 12 cycles of the respective frequency.

Let us consider a circular, passive Sagnac interferometer with radius R. If two signals are sent around in opposite direction, they will arrive at the point of origin after time t,

$$t = 2\pi R/c. \tag{2.2}$$

If the interferometer is rotated with constant velocity Ω, then the signal in the direction of rotation will travel a longer way and will thus exhibit a larger time of circulation t_+. Contrary, the signal going around against the sense of rotation will exhibit a shortened time of circulation t_-,

$$2\pi R \pm x_\pm = ct_\pm.$$

$x_\pm$ is the change of distance the two signals experience depending on the rotational velocity,

$$x_\pm = R\Omega t_\pm$$

where Ω is the rotational velocity, measured in rad/s.

From the two equations above follows

$$t_\pm = \frac{2\pi R}{c \mp R\Omega}. \tag{2.3}$$

[6]Source: http://en.wikipedia.org/wiki/File:Ring_laser_interferometry_shift.png, 11.07.2012

We can calculate the difference in times of circulation

$$\begin{aligned}\Delta t = t_+ - t_- &= \frac{2\pi R}{c - R\Omega} - \frac{2\pi R}{c + R\Omega} \\ &= \frac{2\pi R(c + R\Omega)}{(c - R\Omega)(c + R\Omega)} - \frac{2\pi R(c - R\Omega)}{(c - R\Omega)(c + R\Omega)} \\ &= \frac{4\pi R^2\Omega}{c^2 - R^2\Omega^2} \approx \frac{4\pi R^2}{c^2} \cdot \Omega. \end{aligned} \tag{2.4}$$

Inserting the area of a circle $A = \pi R^2$ gives

$$\Delta t = \frac{4A}{c^2} \cdot \Omega, \tag{2.5}$$

likewise, for the difference of the pathlengths of the two signals

$$\Delta L = \frac{4A}{c} \cdot \Omega. \tag{2.6}$$

Using general relativity, one can show that equations (2.5) and (2.6) are only depending on the area and not the geometry of the resonator.

Let us now consider the active ring resonator. Here, it holds

$$m\lambda_\pm = L_\pm \tag{2.7}$$

where the mode number m remains constant even under rotation.

Eq. (2.7) can be rewritten as

$$\nu_\pm = \frac{mc}{L_\pm}. \tag{2.8}$$

A change of wavelength therefore leads to a beat frequency $\Delta\nu$, which for small changes of length ΔL is given by [1, S. 138]

$$\frac{\Delta\nu}{\nu} = \frac{\Delta L}{L_+ L_-} L \approx \frac{\Delta L}{L}. \tag{2.9}$$

As ν is very large in the optical range (of the order of 10^{14} Hz), even a very small change of wavelength will lead to a measurable beat frequency.

From (2.6) and (2.9) we determine the beat frequency to be

$$\Delta\nu = \frac{4A}{L\lambda}\Omega. \tag{2.10}$$

In (2.10) we can see that the relation between beat frequency and rotational velocity is linear. The proportionality factor increases with increasing resonator area and decreasing circumference. To make sure that an error in measuring the beat frequency will result in an error in the rotational velocity that is as small as possible, the proportionality factor should be maximised. The precision of a laser gyroscope thus increases with the resonator area or if the area is given, is largest for the smallest possible circumference (*i.e.*, a circle).

2.5 Lock-In Effect

Below a certain rotational velocity, the two signals do not exhibit frequencies different from each other. The reason for this are imperfections of the mirror surfaces, which leads to scattering. A small part of the light will be scattered exactly in the direction of the other beam, leading to a coupling of the cw and ccw modes.

The lock-in threshold can be approximated as [1, S.153]:

$$\Omega_L \approx \frac{c\lambda^2 r_s}{32\pi A d} \tag{2.11}$$

where A is the area enclosed by the resonator, r_s the backscattering coefficient and d the beam diameter. For the mirrors used a backscattering coefficient of $r_s \approx 10^{-2}$ can be assumed.

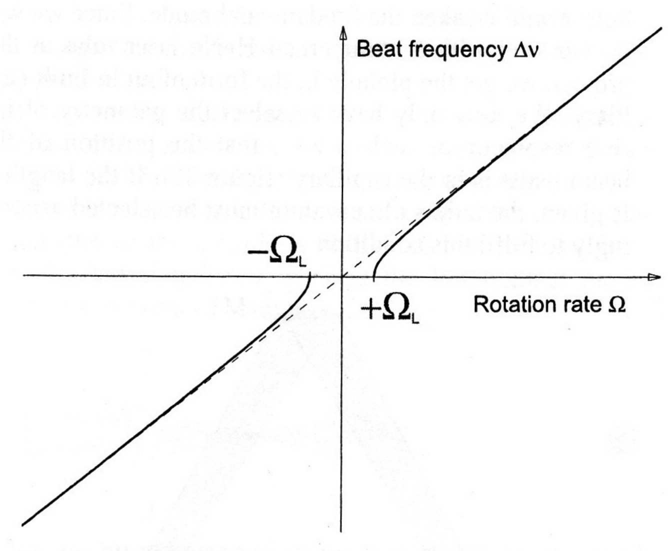

Figure 8: Illustration of the effects of the lock-in threshold. For comparison an ideal case without lock-in effect (dashed line). [2]

Figure 8 shows the influence of the lock-in effect. For rotational velocities below the lock-in threshold $\Omega < \Omega_L$, no beat frequency is detectable. For $\Omega > \Omega_L$ the relation between beat frequency and rotational velocity is non-linear. For $\Omega >> \Omega_L$ it is linear according to eq. (2.10).

2.6 Fizeau Effect

The Fizeau effect describes the change of the velocity of light in a medium as a result of movement of the medium relative to the observer. In a ring resonator, this effect can be observed if the pump energy is delivered through a DC electric field and thus electrons flowing constantly in one and the same direction. This leads to a slightly lower speed of light for the beam propagating against the direction of electron flow slightly higher speed of light for the beam propagating with the direction of electron flow as opposed to if the electrons were stationary.

Electrons flow from cathode to anode. In a gas ring laser, the Fizeau effect therefore leads to different measurements in cw and ccw direction (for the same rotational velocity), depending on how the tube is oriented.

3 Experimental Setup

3.1 Setup of the Laser Gyroscope

The setup of the gyroscope consists of a triangular resonator and a setup of optical elements and detectors behind the outcoupling mirror.

Resonator

The ring resonator consists of a laser tube filled with a helium-neon mixture. To make the resonator lase, the mirrors must be precisely aligned (more on this in chapter 4). To help with the alignment, an alignment laser is mounted outside the resonator. This allows sending a beam in ccw direction around the resonator, and helps with aligning the components. The resonator setup consists of a laser tube as well as two plane and one curved mirror with radius of curvature $R_c = 0.75$ m (see figure 9).

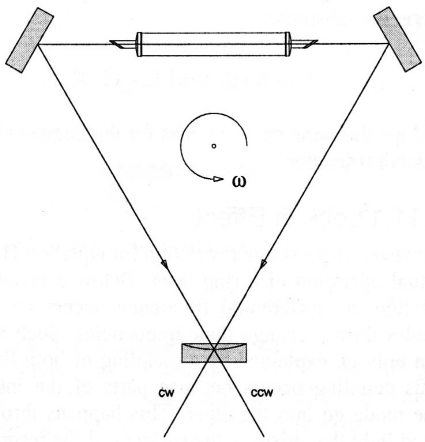

Figure 9: Setup of the resonator. [2]

Because the angle of incidence is about 30°, the effective mirror focal length f* (effective radius R*) is smaller than the actual focal length or radius. To calculate the stability criterion correctly, one needs to account for this by correcting the radius of curvature according to $f^* \approx 0.82f$ or $R^* \approx 0.82R_c$. Figure 10 shows the effect of an angle of incidence of 30° on the focal length of a curved mirror.

Interference system

The setup of the interference system is designed so that the two beams (cw and ccw) cross at a right angle in the center of the beam splitter cube (fig. 11). The beam splitter transmits 50% of the light and reflects the other 50%. The two outgoing beams are thus consisting of 50% cw and 50% ccw beam each. These two beams are then measured using photodiodes.

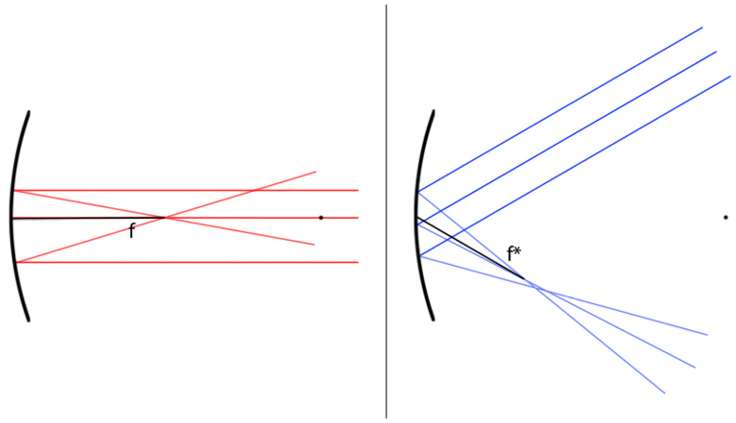

Figure 10: Focal length of the curved mirror for a beam incidence angle of 0° (red) and for a beam incidence angle of 30° (blue).

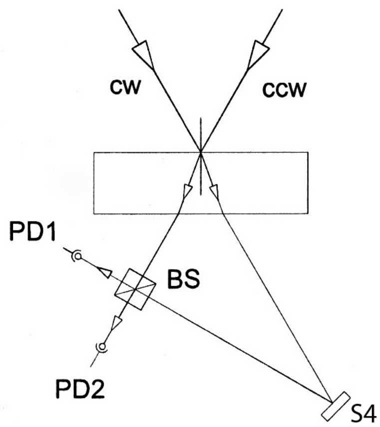

Figure 11: Setup of the interference system. [2]

Mirrors

The used plane mirrors (two mirrors as part of the resonator and one mirror as part of the interference system) are silver-coated mirrors with a transmission rate (for $\lambda = 633$ nm) of 0.2% at an angle of incidence of 30°.
The spherical mirror is also silver-coated with a transmission rate (again for $\lambda = 633$ nm and an angle of incidence of 30°) of 3.3%.

3.2 Setup of Measurement and Control Units

The output signals of the two photodiodes are sent to a differential amplifier, where the difference of the two signals is calculated and amplified. The frequency of the resulting signal is then measured using an oscilloscope. The calculation of the difference does not alter the frequency, but is done to compensate for fluctuations in laser power output (as both signals will experience the exact same fluctuations, the difference will be free of them). In addition, two speakers are driven from the differential amplifier, giving an acoustic feedback of the rotational velocity if desired.

The gyroscope is mounted on an aluminium plate rotated by a stepper motor. The stepper motor is controlled via a LabView application. From the home position, the gyroscope can be rotated by 360° in either direction, *i.e.*, it possesses a total range of rotation of 720°.

The whole experiment can be conducted via a single LabView application (see fig. 12). This application controls the stepper motor and allows measuring the beat frequency. For the measurement of the beat frequency, two methods are provided: A multi-measurement, taking 200 single frequency measurements within a few seconds and then calculating the mean and standard deviation, and a single-measurement, taking a single frequency measurement. The multi-measurement provides better results in general, but sometimes fails to do so especially for very low rotational velocities.

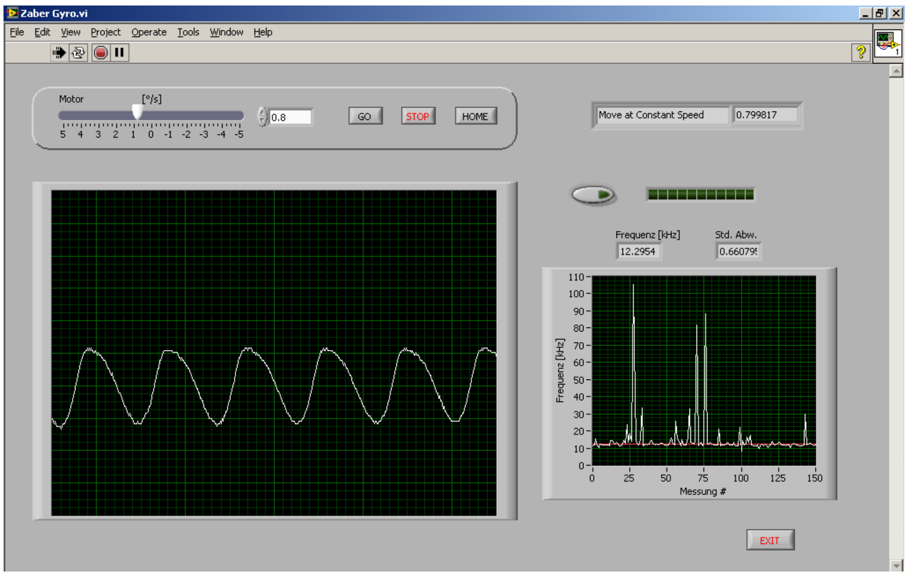

Figure 12: The LabView application allows controlling the stepper motor as well as measuring the beat frequency.

3.3 Error Sources

Possible error sources include the already discussed effects (Fizeau, lock-in, common ground) as well as light present in the room and frequency instability of the laser.
While effects due to the ambient light could be minimised by turning off the lights, no negative impact on the measurements could be found with the lights on. The laser used however is not frequency-stabilised, which means that laser frequency constantly fluctuates. This leads to some unstable or even unusable measurements, especially for small rotational velocities. If the gyroscope is completely still, the signal can be seen to fluctuate by the continuous "twitching" of the output graph. Figure 13 shows an overview of the most important possible error sources.

Errors introduced by the gain medium itself were not discussed up to now. However, because of its refractive index n, the gain medium changes the effective length of the resonator: $L^* = L + d\,(n-1)$, where d is the length of the gain medium. Due to this effect, the laser modes are slightly shifted compared to those of the passive resonator ("mode pulling", see fig. 13 d).

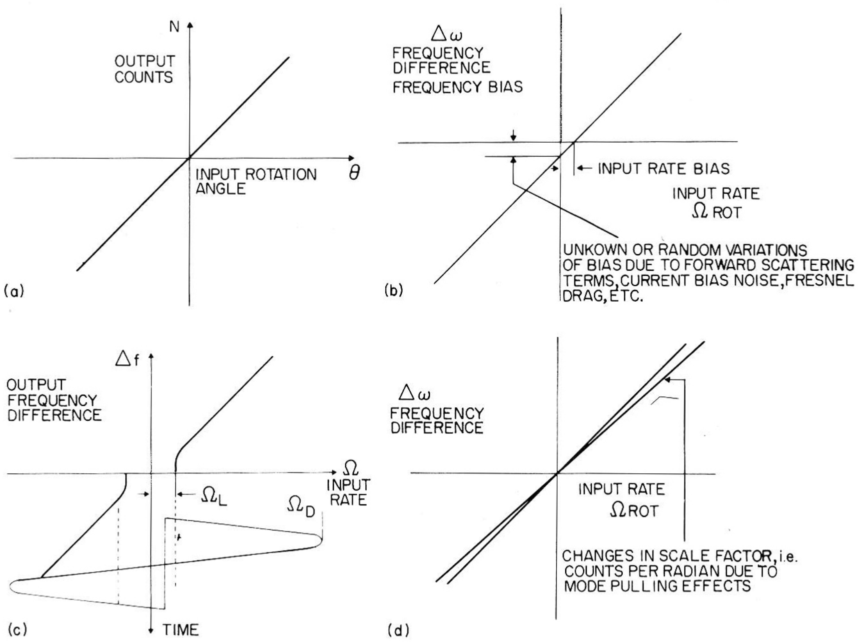

Figure 13: Error sources in a laser gyroscope. [1] a) ideal gyroscope b) Fizeau effect (and others) c) lock-in effect d) mode-pulling effect.

4 Alignment of Optical Elements

Aligning the optical elements must be done with care, as the HeNe laser will not work unless the alignment is very precise. The following instructions should help getting a good alignment, but some patience and trial-and-error is required. It is important that one only continues to the next step once the former is completed.

First check if all parts are at hand:

- 1 HeNe laser with adjustable holder (already mounted)
- 1 diode laser with collimation optics (already mounted)
- 3 plane mirrors with mounts
- 1 spherical mirror with mount
- 1 beam splitter cube with mount
- 2 photodiodes with mounts
- 1 imaging lens with raising adapter

4.1 Resonator Alignment

The resonator setup consists of the laser tube, a curved and two plane mirrors. To align the mirrors, the alignment laser is coupled into the resonator. The mirrors are then adjusted until the alignment laser interferes with itself after a full revolution.

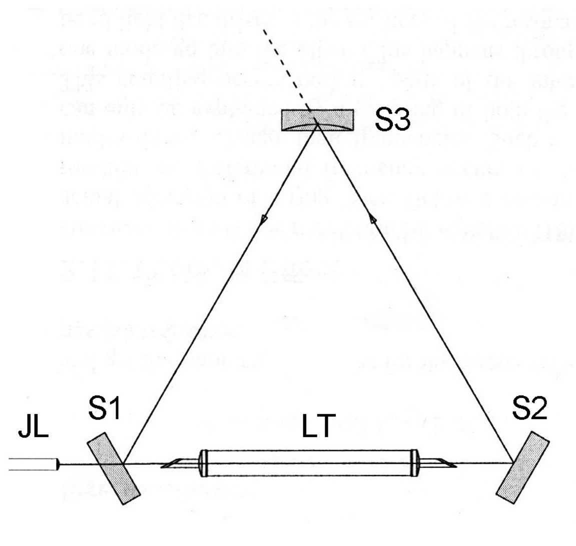

Figure 14: Setup of the laser resonator consisting of two plane mirrors (S1,S2), one spherical mirror (S3), the laser tube (LT) and the alignment laser (JL). [2]

- Start by setting up the two plane mirrors, the curved mirror and the laser tube according to figure 14. Make sure the resonator length is within the stability limits. Turn on the alignment laser. Now adjust the laser tube (LT) until the beam from the adjustment laser (JL) goes exactly through the laser tube's capillary. Use the intensity and geometry of the beam coming out of the other side. For this purpose, exchange mirror S2 with the imaging lens and put a screen (paper) behind. Turn off the lights if needed.

- Put S2 back and adjust it, so that the beam hits the center of the curved mirror S3. Adjust S3 so that the beam hits S1 exactly where the beam passes through the first time.
- By adjusting S1 the beam must now be directed through the tube again. This part is very tedious because the beam can hardly be seen after the second pass through the tube. Put the imaging lens behind S2 and project the beam onto a white screen or wall. By manipulating S1 and S3 the secondary beam can be adjusted until some interference (flickering) can be seen on the screen. By careful adjustment of the same two mirrors this interference pattern must improved until it is roughly concentric (this is very hard to discern). It is helpful, if S1 and S3 are only adjusted either horizontally or vertically at a time. The mirrors should be adjusted in the direction that makes the distance between maxima (minima) of the interference pattern larger. Note that the radius of the interference pattern is small and it is partially covered by the laser spot, interference rings far outside the central spot are due to the interference of scattered light and not of the two primary beams!
- As soon as the interference pattern behind S2 is optimized, place the screen behind S3. You should see interference rings there as well, if not, repeat the previous step. Once you see a vaguely circular interference flickering behind S3, turn on the HeNe laser tube. If the cavity starts resonating, you will see a circular (or slightly oval) red dot in close proximity to the interference. This is however, rarely the case right away. Most likely, you will need to apply *minimal* corrections to mirrors S1 and S3 until this happens. Once the red dot of the HeNe laser is visible, the alignment laser can be turned off. Now, use S1 and S2 to maximise the intensity. You can also adjust S3, but be careful as its mount is very sensitive and you may quickly lose the beam altogether.

4.2 Interference Alignment

- Position mirror S4 so that the two outgoing beams cross at a right angle (as depicted in figure 11).
- Position the beam splitter so that the two beams cross at the very center of the cube and both beams enter the cube at a normal angle with its faces.
- Now adjust the positions of mirror S4 and beam splitter so that the two beams overlap. Position the imaging lens behind the beam splitter. Now start shifting both the beam splitter and the mirror simultaneously (in the same plane), so that the beams constantly overlap. You should see

an interference pattern emerge. Adjust both horizontal and vertical plane until you see a fully circular interference. The beam should seem to disappear and reappear in full if you shift the gyroscope slightly (as opposed to having multiple rings).

5 Theoretical Assignments

1. Calculate upper and lower boundaries for the resonator length so that the stability criterion is fulfilled.
2. Why should the resonator length be as large as possible? Is there any reason for a smaller resonator length?
3. Calculate the lock-in frequency of the setup (only possible once you have the resonator set up).
4. Estimate the proportionality factor of the linear relation between rotational velocity and beat frequency.

6 Experimental Assignments

1. Set up the resonator and get it to lase.
2. Measure the beat frequency $\Delta\nu$ as a function of the rotational velocity Ω. Make sure that there are no other experiments being conducted at the same time, or the oscilloscope measurement will be disrupted by interference.
3. Plot the measured beat frequencies against the rotational velocity. Fit a linear regression to calculate the proportionality factor and compare this to the theoretical result. Fit the linear regression weighted by the inverse of the variance (obtained from LabView) to account for the measurement errors. Using MatLab's 'fit' function is the easiest way to implement this.
4. Explain the influence of the Fizeau effect with the help of the curve.
5. Can you observe the lock-in effect in your data?

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