Elementarteilchenphysik

Antonio Ereditato

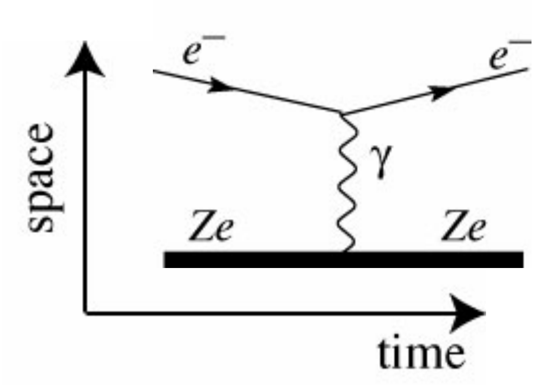
LHEP University of Bern

QED and Feynman graphs

QED, a Quantum Field Theory

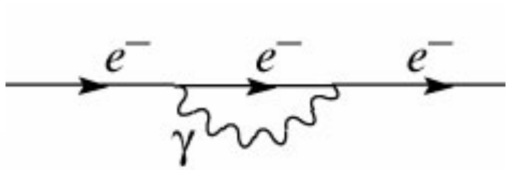
Quantum Electro Dynamics (QED): the first QFT, developed in the 50's by Stueckelberg, Schwinger, Tomonaga and Feynman

Hydrogen atom: the proton electric field felt by the electron is quantized, a quantum system of photons that interact with the electric charges (anticipate Feynman graph notation):



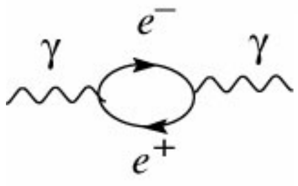
The electron is scattered, the heavier proton virtually does not move

An electron propagating in vacuum might undergo this process:



(vacuum is not really "empty").

Also the propagating photon is "not only" a photon…



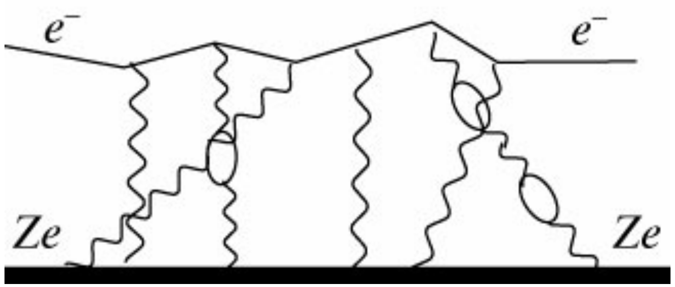
In order to conserve the energy, photon and electron pair emission and re-absorption must occur for a time interval such that:

$$\Delta t \leq \frac{\hbar}{\Delta E}$$

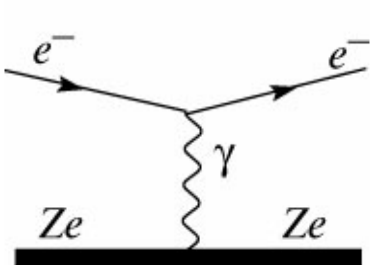
Difference with the “classical” interpretation: the electron interacts with the proton field and also with its own field (self interaction). This creates “infinities” in the theory, since part of the electron mass-energy goes into photons or virtual electron loops. The electron ***e/m*** ratio increases. This is solved by “renormalizing” the electron (in vacuum) mass to the experimentally measured at low energy (see later).

A bound electron (hydrogen atom) moves around its “nominal” position and looks like a charged sphere of ~10^{-15} m radius due to the “vacuum” particles around it. Its increased energy creates levels splitting; this is the case of the Lamb shift between the orbitals $^2S_{1/2}$ and $^2P_{1/2}$, otherwise with the same energy.

In addition, with some probability, the hydrogen atom could look like as composed of 2 electrons and 1 positron (plus the proton) or even including 3 e^- and 2 e^+:



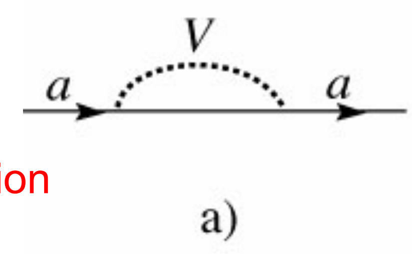
In QED both interaction and particles are described by quantum fields.

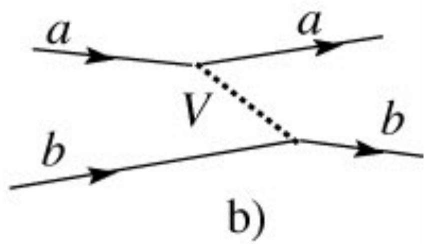


In QED, the initial electron is destroyed by an annihilation operator. After, a creation operator creates the final state electron.

Interaction: exchange of bosons

Emitted bosons by a can either be reabsorbed by a or absorbed by b.
In the second case we talk of an interaction





In either case: $\Delta t \leq \dfrac{\hbar}{\Delta E}$

The distance travelled by the boson is $R = c\Delta t$ for a range of the interaction: $R = c\Delta t = c\hbar / m$

The graph b) identifies the amplitude of the scattering. It contains:

1) the probability to emit the boson V,
2) its propagation from a and b,
3) the absorption of the boson V.

Consider now a non-relativistic scattering (*e.g*. Rutherford scattering) from a central potential

The propagator

The scattering amplitude $f(\mathbf{q})$ corresponds to the potential $U(\mathbf{r})$ (Fourier transform)

$$f(q) = g_0 \int U(r) e^{i\mathbf{qr}} \, dV$$

For a central potential: $U(\mathbf{r}) = U(r)$ *and* one can integrate by setting:

$$dV = r^2 dr \sin\theta d\theta d\phi$$

$$\mathbf{qr} = qr\cos\theta$$

As an example, we introduce the Yukawa potential $U(r)$ and obtain a relation describing the potential in momentum coordinates:

$$f(q^2) = g_0 \iiint U(r) e^{iqr\cos\theta} \sin\theta d\theta d\phi r^2 dr$$

$$= g_0 \iiint \frac{g}{4\pi r} e^{-rm} e^{iqr\cos\theta} \sin\theta d\theta d\phi r^2 dr$$

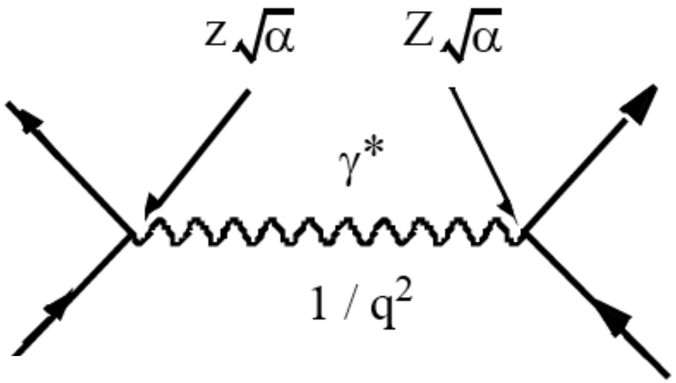
$$= g_0 \frac{1}{q^2 + m^2} g$$

i.e. the scattering of a particle of coupling g_0 by the static potential generated by a massive source of strength g. In the general case also energy ΔE is transferred with momentum: $\Delta\mathbf{p} = \mathbf{q}$ (4-momentum transfer): $q^2 = \Delta\mathbf{p}^2 - \Delta E^2$ (relativistically invariant)

Therefore, the amplitude (matrix element) for a scattering process is the product of two coupling constants (of the boson with scattered and scattering particles) multiplied by a propagator term:

For an electromagnetic process, the coupling constant is the **electric charge**:

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2}{4\pi} = \frac{1}{137} \quad (\hbar = c = \varepsilon_0 = 1)$$



$$f(q^2) = z\sqrt{\alpha} \cdot \frac{1}{q^2} \cdot Z\sqrt{\alpha}$$

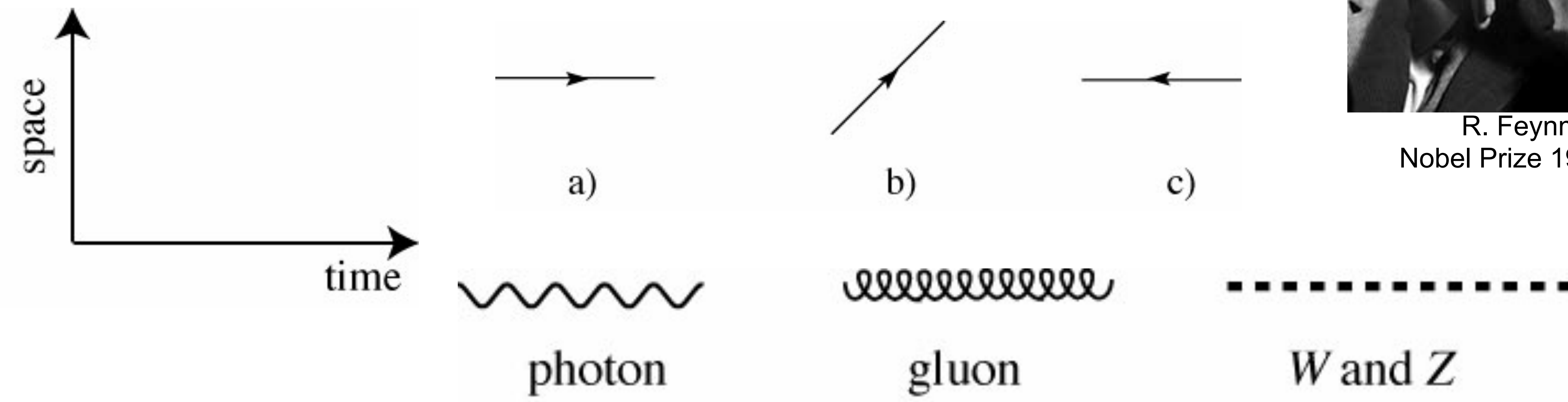
Reminder: the rate W of a given reaction (collision, decay) is given by $|f(q^2)|^2$ multiplied by the phase space (Fermi Golden rule):

$$W = \frac{2\pi}{\hbar} |M|^2 \int \rho_f \, d\Omega$$

Quantum Field Theory, QED and Feynman graphs

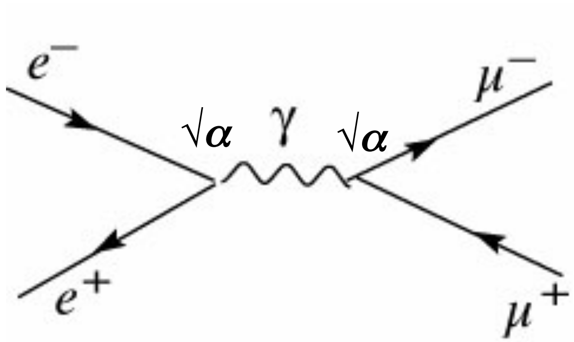
Feynman graphs describe mathematical expressions but also graphical and intuitive representations of particle interactions.

R. Feynman
Nobel Prize 1965

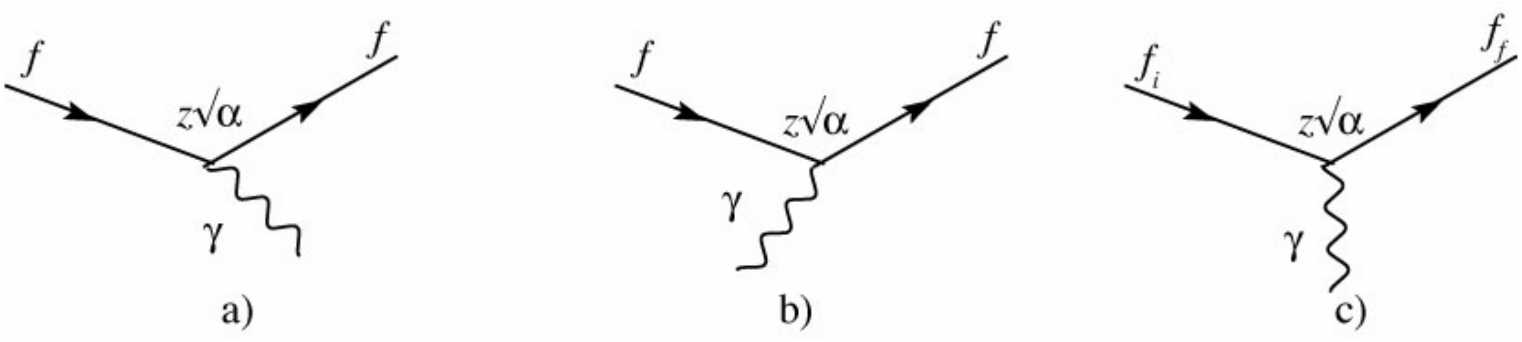


Practical rules:

1) Incoming/outgoing particles/antiparticles
2) Vertices
3) Real and virtual particles



QED basic graphs



The initial fermion disappears at the vertex.
A photon and another fermion appear.

An initial fermion and a photon disappear. Another fermion appears.

This graph is equivalent to the previous ones. We just add the indexes i and f

The vertex expresses the interaction Hamiltonian:

$$z\sqrt{\alpha}A_\mu \bar{f}\gamma^\mu f$$

f and $\bar{f}$ are Dirac b-spinors. The first destroys the initial fermion. The second creates the final fermion

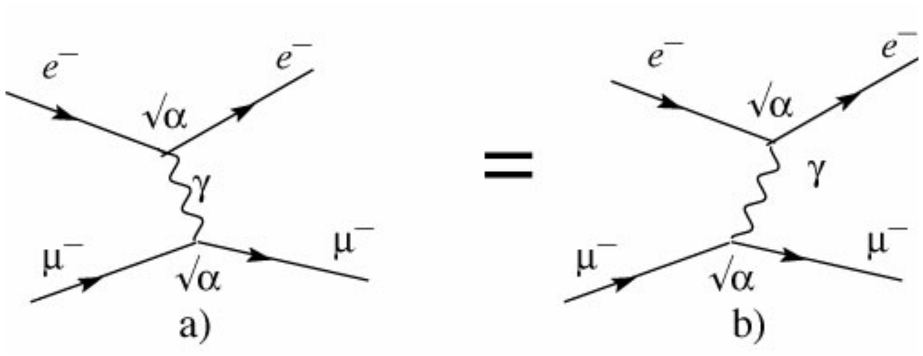
A_μ Is the quantum analogous of the classical potential

$\bar{f}\gamma^\mu f$ Is called “electromagnetic current”

Take, as an example, the elastic scattering between an electron and a muon

$$e^- + \mu^- \rightarrow e^- + \mu^-$$

The first order graph is:

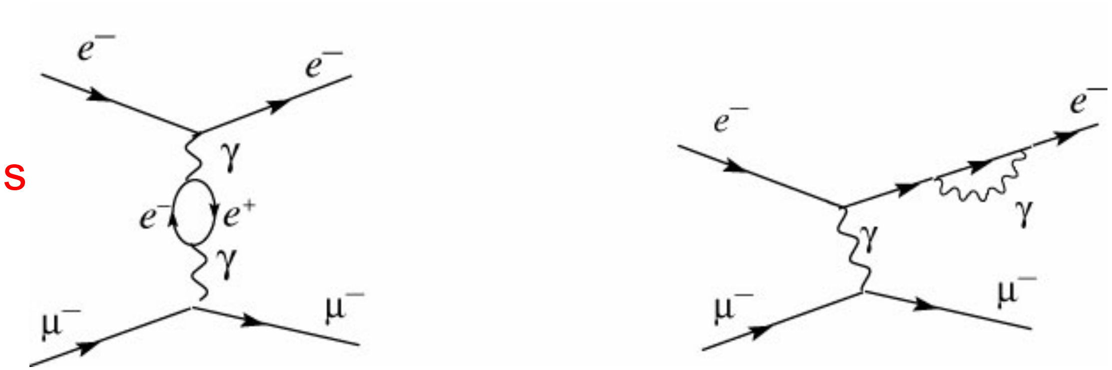


The amplitude of the process is proportional to the product of the two vertex factors:

$$(\sqrt{\alpha} A_\mu \bar{e} \gamma^\mu e)(\sqrt{\alpha} A_\mu \bar{\mu} \gamma^\mu \mu)$$

Emission/absorption amplitudes are proportional to e, and hence to $\sqrt{\alpha}$, the scattering amplitude to $\sqrt{\alpha} \cdot \sqrt{\alpha} = \alpha$, and the rate/cross section to the amplitude squared, *i.e.* α^2

Higher order contributing graphs to the same reaction:



More on antimatter (particles and antiparticles)

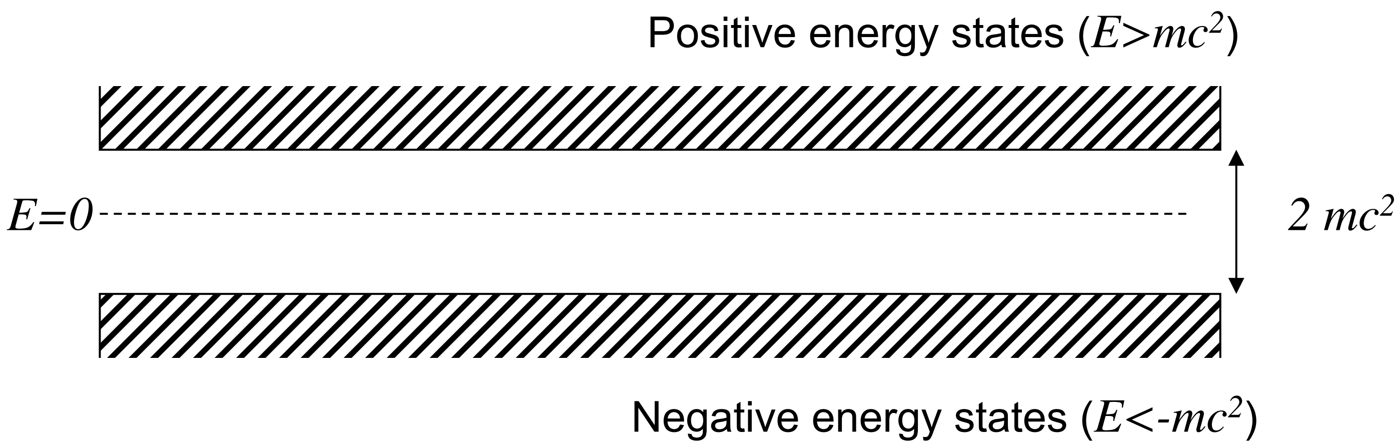
Dirac postulated in 1931 (before the discovery of the positron) the concept of antiparticles: same mass but opposite electric charge and magnetic moment.

Energy-momentum relation implies that negative energy solutions are in principle "allowed":

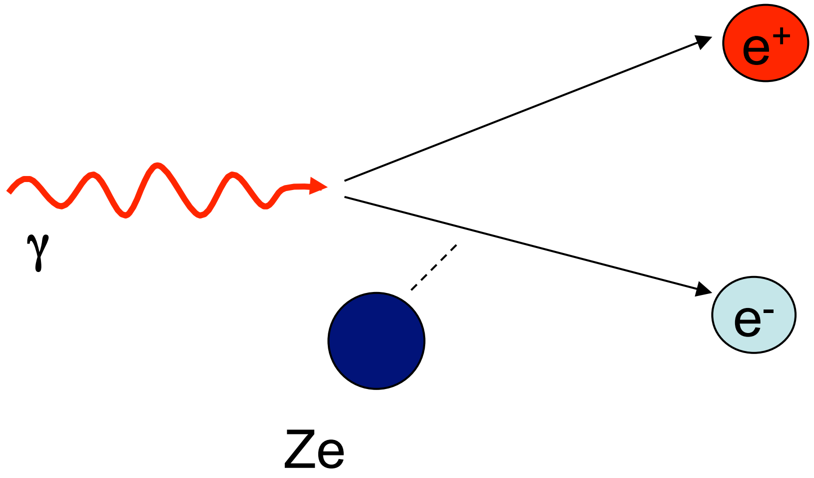
$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

Interpreted as particles with $-E$ and $-p$ traveling backward in time, in turn equivalent to antiparticles traveling forward in time with positive energy (e.g. the positron).

Dirac: vacuum is an infinite deep sea filled with negative energy states electrons. Positive energy electrons cannot fall in the sea (Pauli principle). If we supply an energy $E>2m_e$ an electron can jump in the positive energy levels leaving a hole (*i.e.* a positron). The situation is different for bosons.



Pair production: $\gamma \rightarrow e^+ + e^-$



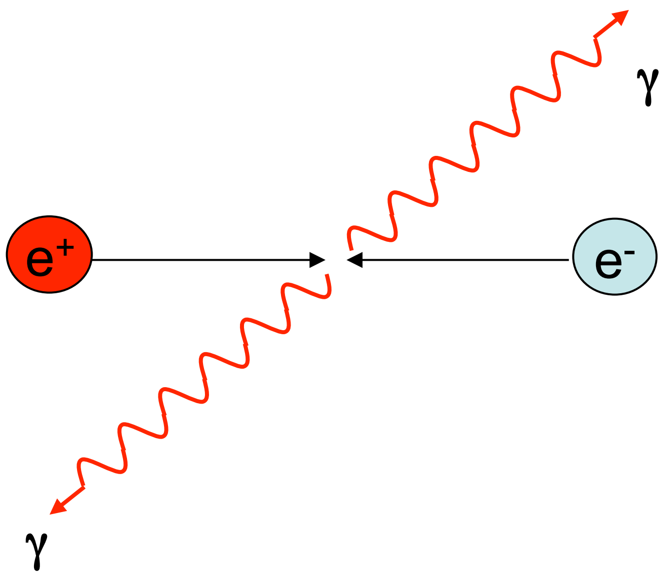
Need:

$E_\gamma > 2\ m_e$

"spectator" nucleus to conserve momentum

Electron-positron annihilation:

$e^+ + e^- \rightarrow \gamma + \gamma$



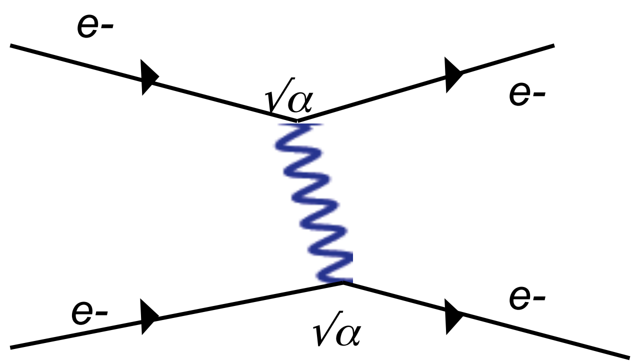
Two back-to-back γ each with 1/2 of the total available energy

Notable QED reactions

Coulomb scattering between two electrons:
amplitude proportional to $\sqrt{\alpha}$ x $\sqrt{\alpha} = \alpha$

The virtual photon has a "mass": $m^2 = -q^2$
(propagator $1/q^2$)

Matrix element proportional to α/q^2
Cross section proportional to α^2/q^4



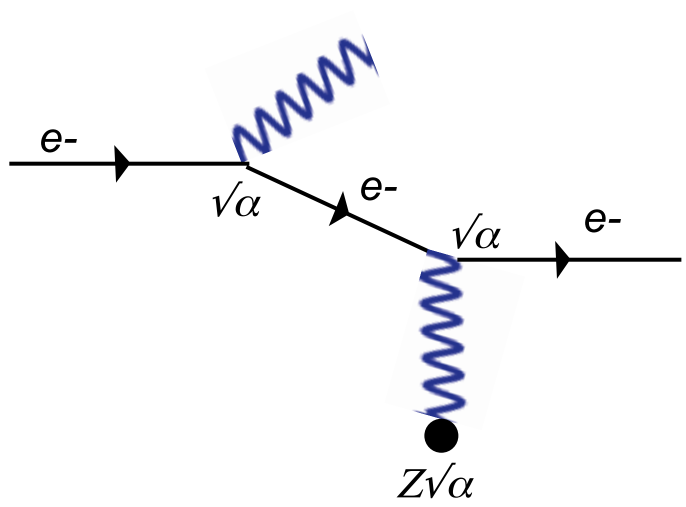
Leading order processes (in α)

Photon Bremsstrahlung: electron emitting a real photon when accelerated in the Coulomb field of a nucleus.

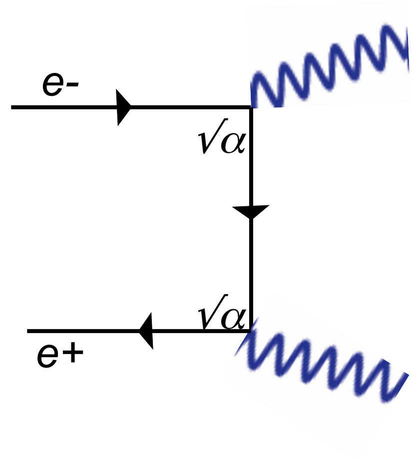
The electron in the second line is virtual (off mass shell)

A virtual photon is exchanged with the nucleus to conserve the momentum

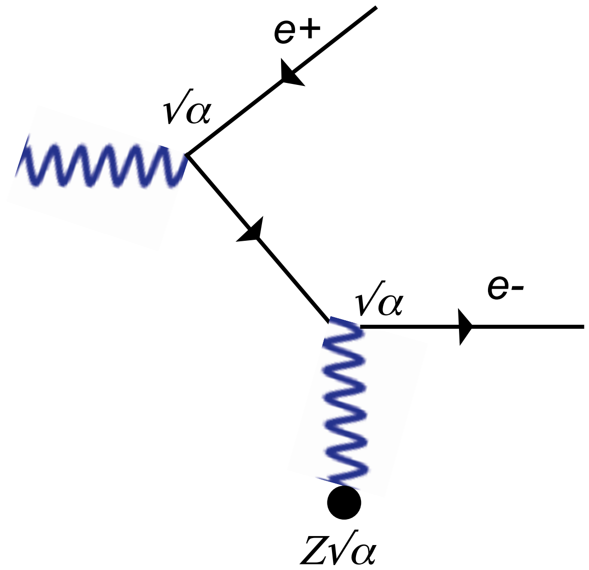
Cross section proportional to α^3



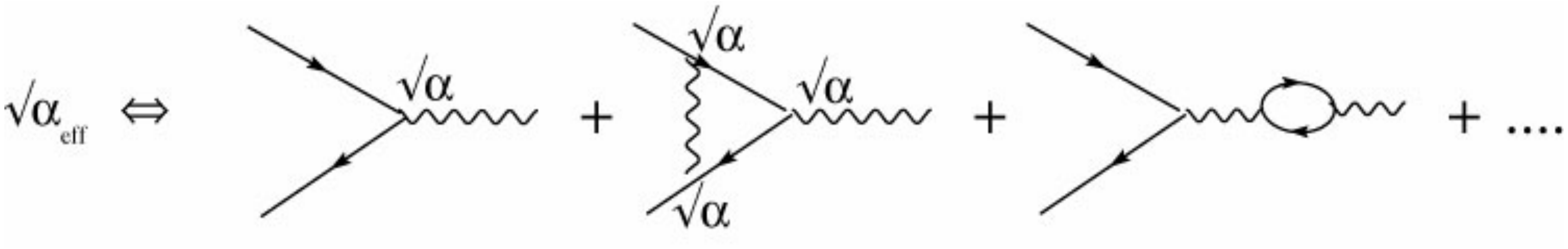
e+e- annihilation



Pair production

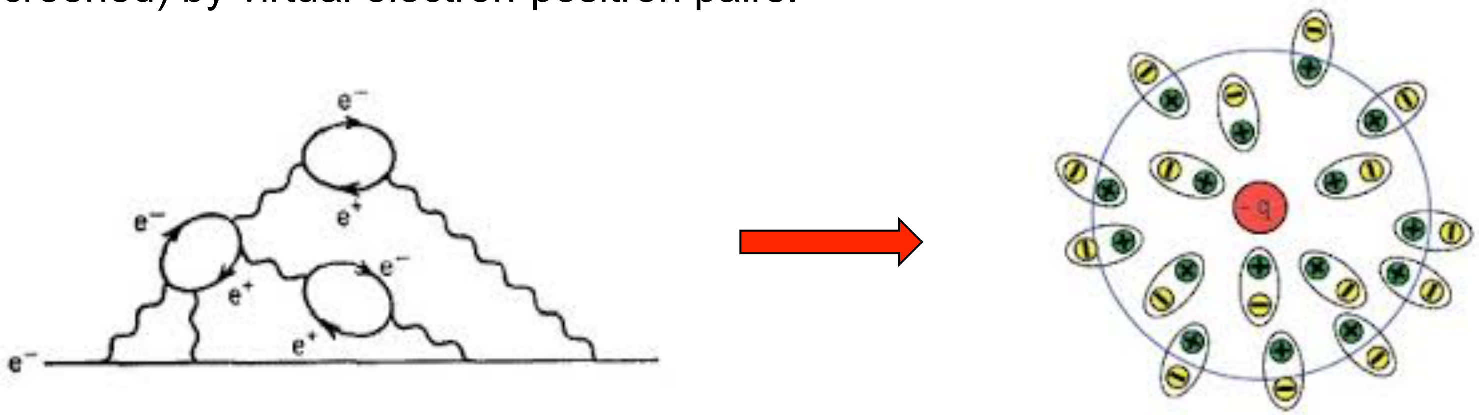


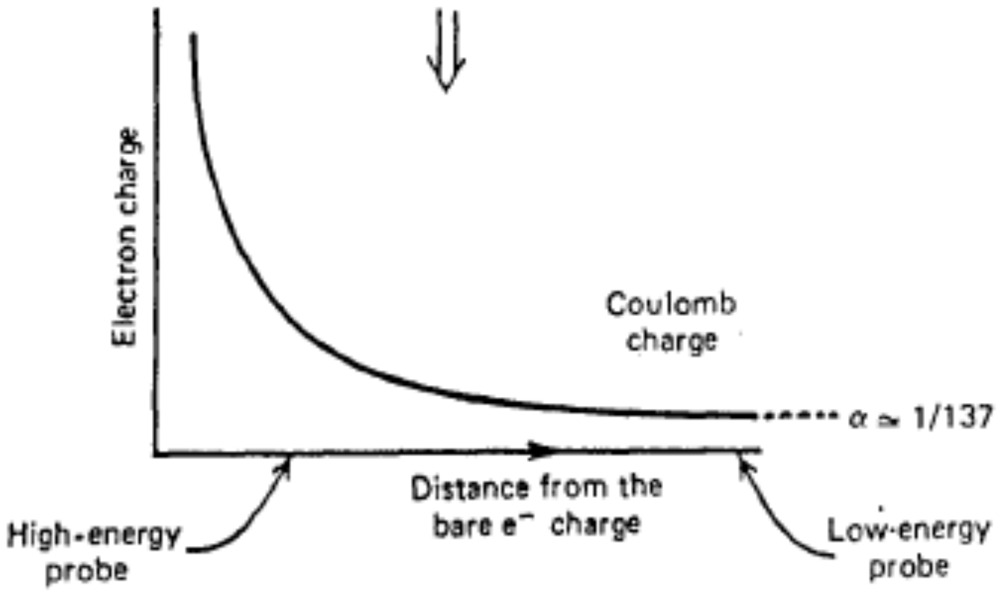
The “bare” electron charge (present at any vertex) is virtually infinite. But we do not observe it. The experimental $\sqrt{\alpha}$ (electron charge) is the “effective” one, which includes all the (infinite) graphs that can occur at any order:



Renormalization: replace the un-measurable bare quantities m_0 and e_0 with the experimentally measured values for m and e.

Higher order means higher energy/closer distance. The bare charge appears then surrounded (and screened) by virtual electron-positron pairs.





Consequence from the renormalization: α is not a constant but “runs” with the energy:

α = 1/137 (low energy) 1/128 at ~100 GeV

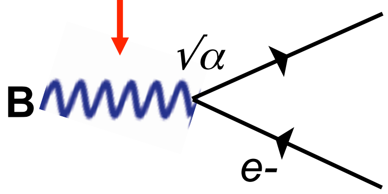
Renormalization can only be applied to “gauge invariant theories” (such as QED).
Examples are:

In electrostatics the potential can be re-defined without modifying the physics.

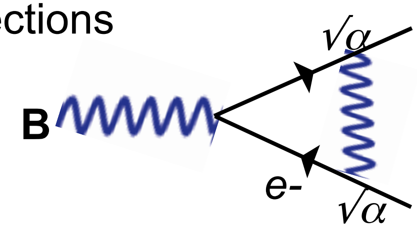
In QM the phase of a wave-function can be changed without affecting the observables.

Example: self-energy and electron magnetic moment

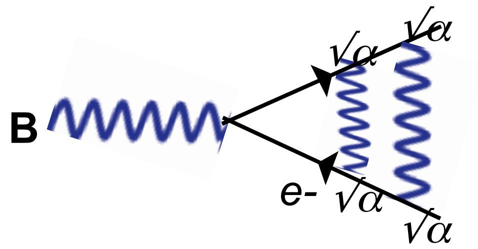
At the leading order term: μ_e is equal to the Bohr magneton (Dirac) $\mu_B = \dfrac{e\hbar}{2m_e c}$



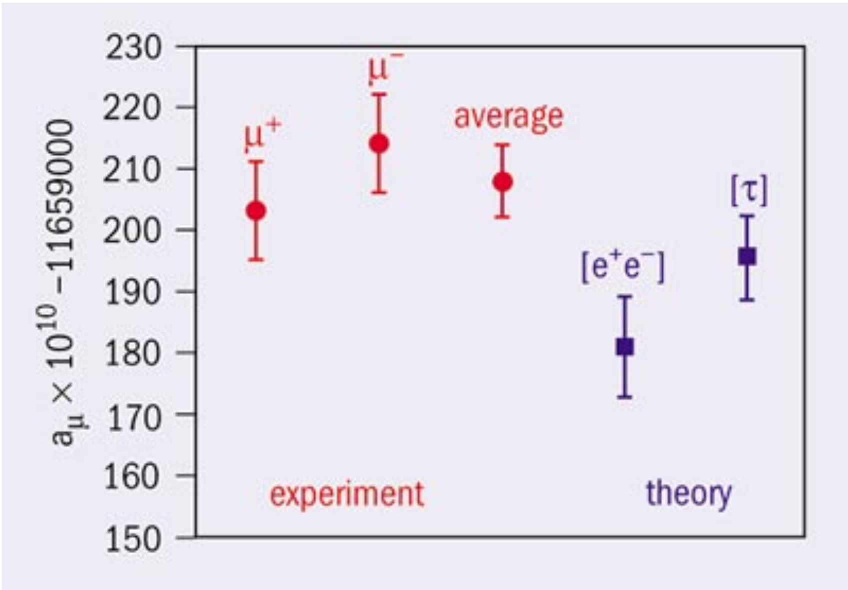
There are, however, higher order corrections



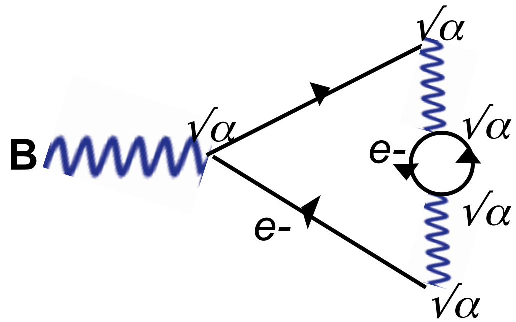
For next-to-leading graphs, the (bare) electron charge still stays with the electron and the field will interact with it, but part the electron mass-energy goes into photons or virtual electron loops. The ***e/m*** ratio slightly increases.



If we define μ_e equal to $g\mu_B s$ (with ***s*** the electron spin ±1/2) the difference between the actual magnetic moment, including higher order contributions, and the Bohr magneton is expressed by the deviation of ***g*** from ***2***:



$$\left(\frac{g-2}{2}\right)^{theory} = O(10^{-3})$$



Astonishing agreement with the experimental results!
Indication of disagreement at the level of <1/10⁸