Institute of Applied Physics
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LABCOURSE

Diffraction of light by ultrasonic waves

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September 2014

The goal of this lab course is to get familiar with the physical principle of ultrasonic waves and its interaction with light. It will be investigated how light diffracts in a transparent medium, which is subjected to periodic density variations, and how this can be used to measure the speed of sound (SOS) inside that medium within an accuracy of a few permille. For the implementation of automated data acquisition and image analysis basic knowledge of LabView (Laboratory Virtual Instrumentation Engineering Workbench) and MatLab is required.

1 Theory

1.1 The basics

For a general introduction into the theory behind, it is recommended to read the literature as given in the references. This is in particular required for the following topics: 1. generation of ultrasonic waves using piezoelectric crystals([1], page 49ff), 2. properties of excited piezoelectric crystals (resonance frequency, harmonic frequencies) ([1], page 231-236), 3. optical approaches for the determination of the speed of sound ([1], page 173ff). Reference values for the speed of sound in different liquids in the table ([1], page 263). In addition to the provided references it is strongly advised to perform a literature research (Library, Internet, etc.) for a sufficient preparation.

1.2 Diffraction of light by ultrasonic waves

The interaction of light and sound exhibits many interesting and useful effects. Sound waves can deflect light, change its frequency and modulate its phase and amplitude. In turn the diffracted light can probe the spatial distribution of acoustic energy in the sound beam and give information on the velocity and attenuation of the sound wave and the elastic properties of the material. In 1932 Debye and Sears discovered in the USA and Lucas and Biquard discovered in France that transparent media diffract light when an ultrasound wave is sent trough them. This effect is a consequence of a periodical variation of the refractive index, which in turn is a consequence of a local periodical pressure change caused by the ultrasound wave. This periodic variation coincide with the wavelength of the sound field inside the liquid. The periodic variation in the liquid act as phase grating on the light beam, which incides perpendicular to the propagation direction of the waves, leading to the appearance of diffraction phenomena.

To calculate the interference pattern on the screen we need to apply Huygen's principle. We consider every point on the exit plain x = a after the ultrasound field as a starting point of a spherical wave with an amplitude and phase of the light wave in this point, see Fig. 1. Since the exact solution of the eikonal S in the domain of the ultrasonic wave is very difficult, we would like to use an approximation in which the light rays pass in parallel through the ultrasound field and are modulated only in the local and temporal phase. This corresponds to a linear approximation of the eikonal and amplitude.

$$S = n(y,t)x \text{ and } A = A_0$$

In this approximation all the polarization directions are identical, so we can abandon the vector notation. In region I we have parallel light so

$$E_I = A_0 e^{i(\omega t - k_0 x)} \tag{1}$$

, where k_0 is the wavevector $k_0 = \frac{2\pi}{\lambda_0}$ and n, the index of refraction

In region II, from our assumption

$$E_{II} = A_0 e^{i(\omega t - k_0 n(y,t) * x))} \tag{2}$$

we consider region III as a superposition of spherical waves ΔE_{III}

$$\Delta E_{III} = E_{II} * \frac{1}{R} * e^{i(\omega t - k_0 R))} * \Delta y \tag{3}$$

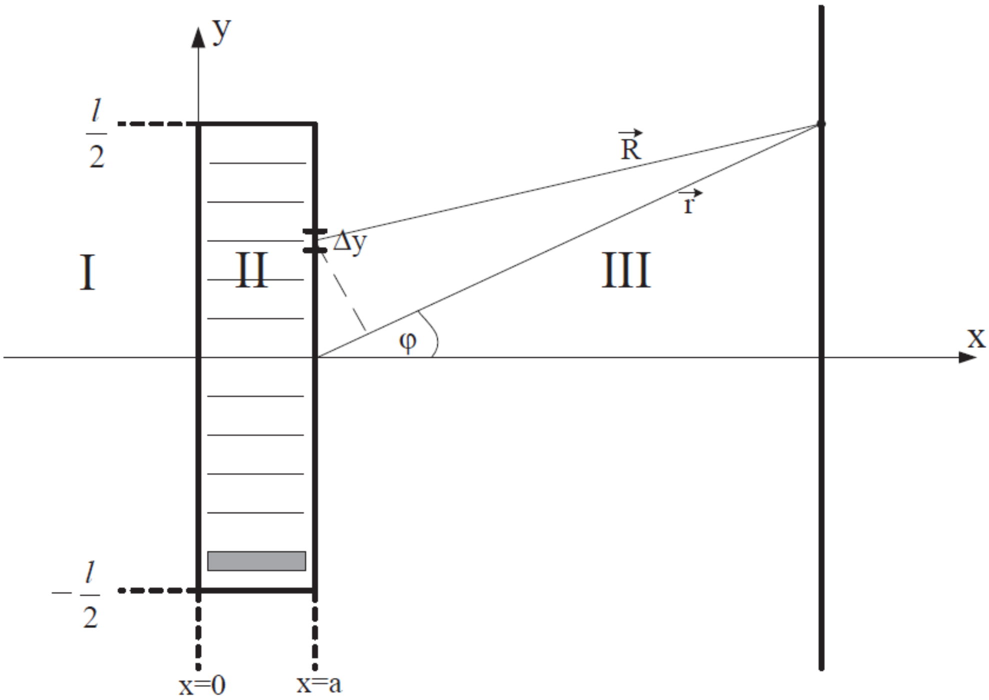

Figure 1: scheme of diffraction principle; I, II and III indicate different regions, before and after the diffraction of the incident planar light waves

For large R or observations around small angles

$$R \cong r - y * sin(\varphi) \tag{4}$$

We assume that the ultrasonic wave propagates in the y-direction, thus

$$n(y,t) = n_0 + \Delta n_0 sin(\Omega t - Ky) \tag{5}$$

where K is the wavenumber K $= \frac{2\pi}{\Lambda}$ and Ω is the oscillation frequency of piezoelectric element.

Together with equation (2), (3) and (4) we get

$$E_{III} = \frac{A_0}{r} e^{i(\omega t - k_0 n_0 a - k_0 r))} \int e^{-i(k_0 \Delta * n_0 * a * sin(\Omega t - Ky)) - k_0 * y * sin(\varphi)} dy \tag{6}$$

After expanding the e^{i*sin}-function into a Fourier series and introducing the Bessel function J, the integral E_{III} can be solved and expressed in terms of intensity:

$$I = \frac{A_0^2 l^2}{r^2} \sum J^2(k_0 \Delta n_0 a) * \left[\frac{sin\left[\frac{l}{2}(k_0 sin(\varphi) - \nu k)\right]}{\frac{l}{2}(k_0 sin(\varphi) - \nu K)} \right]^2 \tag{7}$$

The function $\frac{sin^2 x}{x^2}$ is maximal for $x = 0$ and drops rapidly from each side of $x = 0$. Thus we have strong intensity (interference fringes) around the angles φ_ν in our wave field ($r >> l$). For small angles we obtain

$$sin(\varphi_\nu) = \nu \frac{\lambda_0}{\Lambda} \tag{8}$$

where φ_ν is the angle of the ν-th diffraction maximum, λ is the optical wavelength and Λ is the acoustic wavelength resp. the phase lattice constant.

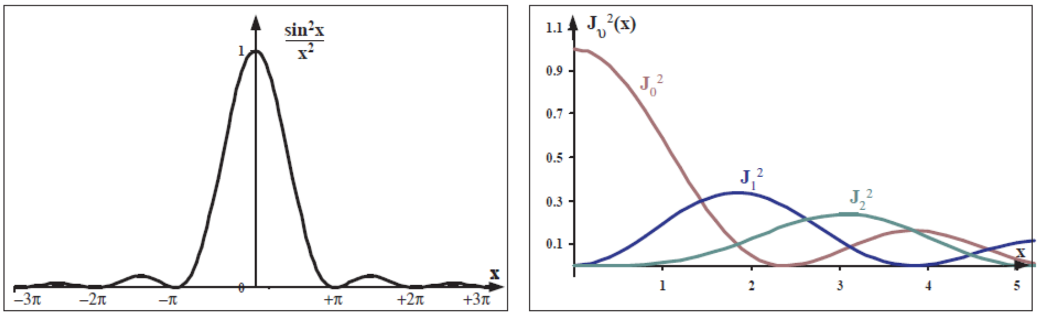

Figure 2: plot of sinc-function; note that the distance of the interference fringes from each other is the same as with a diffraction grating with a lattice constant of the length of the ultrasonic wave

By determining the diffraction angle φ_ν, based on the distances between the different diffraction maximums, we can determine the phase grating lattice constant, which corresponds to the acoustic wavelength. By applying the known relation between speed of sound, acoustic frequency and acoustic wavelength, the speed of sound can be calculated as the following

$$c_{speedofsound} = \Lambda * f_0 = \frac{\nu * \lambda}{sin(\varphi_\nu)} * f_0 \tag{9}$$

where f_0 refers to the excitation frequency of the piezoelectric element.

1.3 Experimental determination of the speed of sound in a liquid using optical diffraction

Making use of the diffraction of monochromatic light with a given wavelength λ by an ultrasonic wave, we can determine the ultrasonic wavelength Λ inside the medium. By measuring the frequency, f_0,of the electric excitation of the piezoelectric crystal, we can determine the speed of sound inside the medium. A helium-neon (He-Ne) laser is used a light source, which provides monochromatic and collimated beam of light with a well-known optical wavelength. For the determination of the observed diffraction angles, precise distance measurements are required. The determination of the excitation frequency, f_0, will be performed using a digital oscilloscope.
It is possible to determine an absolute value of the speed of sound inside the medium with an accuracy of a few permille, when the experiments are performed in an accurate way.

1.4 Temperature dependence of the speed of sound

The measured speed of sound inside the liquid is strongly depending on the temperature of the medium. With the previously described method of determining the speed of sound based on light diffraction, even very small temperature changes and therefore changes in SOS can be detected. During the SOS measurement it is therefore important to measure in addition the temperature of the medium. Single SOS measurements for different temperatures, can be used to determine the temperature coefficient, which allows a scaling of the observed SOS values and therefore a comparison at 20°C.

2 Materials and Methods

2.1 Experimental setup

A scheme of the experimental setup can be seen in Fig. 3. The main components are the following:

- He-Ne Laser as coherent and monochromatic light source (λ= 633nm) (L)
- Lens systems, consisting of an microscope objective and two lenses (O, L_1, L_2)

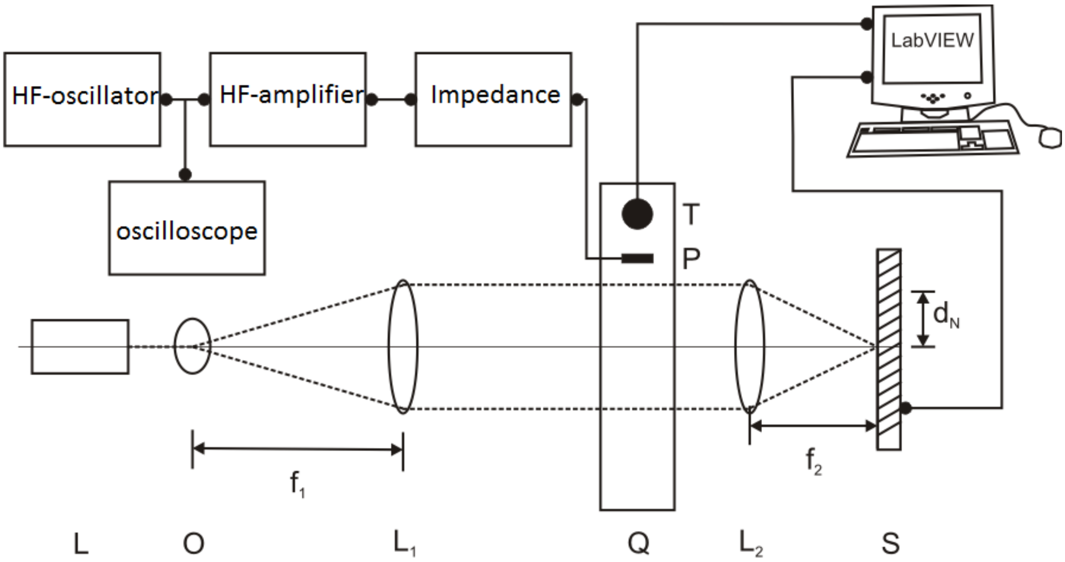

Figure 3: experimental setup: P = piezoelectric crystal with HF-oscillator, oscilloscope to determine excitation frequency, HF-amplifier and Impedance to avoid feedback loops. T = USB-temperature sensor. Arrangement of components for optical diffraction : L = lightsource (He-Ne Laser); O = microscope objective; L_1 = lens (f_1 = 200 mm); Q = glass cuvette with piezoelectric crystal; L_2 = lens (f_2 = 100 mm); S = difffraction pattern on CCD chip (CMOS VGA Chip, OV7720)

- OV7720/OV7221 CMOS VGA (640x480) Camera Chip Sensor (S) and lab computer
  (Datasheet of CMOS VGA sensor see material; pixel size: 6.0μm x 6.0μm)
- Glass cuvette filled with ethanol (Q)
- Piezoelectric element (P)
- Highfrequency (HF) source, consisting of HF-oscillator (Wavetek) and HF-amplifier, control of piezoelectric element
- Oscilloscope
- USB-temperaturesensor GO!Temp (T)
  (Range: -20°C bis 110°C, Resolution: 0.07°C, Error: ±0.5°C)

2.2 Lightsource

As coherent, monochromatic light source a "Uniphase" He-Ne laser is used. It emits light at a wavelength of 633nm and has a average power of around 1 mW. To avoid an overexposure of the CCD ship, different grey-filters are available for optical attenuation.

2.3 Generation of ultrasonic waves

The experiments will be performed in two different liquids: water and ethanol. The ultrasonic waves are generated with an oscillating **piezoelectric element**. It is important that the piezoelectric element is used **only inside the liquid** . Otherwise it will break due to the much smaller damping in air. The thickness of the element is 3.2 mm.

By using of a serie-inductance (grey metal box between piezo element and HF-amplifier) it is possible to adapt the impedance of the amplifier to the impedance of the element.This avoids feedback loops which can harm the HF-amplifier.

2.4 Frequency measurement

The oscillation frequency of the piezo element will be determined with the digital oscilloscope. The resonance frequency of the piezoelectric element can be determined experimentally, when the maximum number of diffraction maximums is observed.

2.5 Diffraction pattern

The diffraction pattern, consisting of equidistant spots of high intensity (diffraction maximums) is projected on the CCD chip which can be read in real-time using the corresponding camera software *PS3.exe*. The corresponding icon can be found on the desktop of the lab computer. This can be very convenient during the alignment of the setup, for real-time feedback. It is recommended to use *Matlab* or *ImageJ* for the image analysis and exact distance determination between the different spots. A more sophisticated way of analysis, which is required for the ethanol measurements and which leads usually to a higher accuracy of the results, is an automated and embedded read out of the camera, combined with the temperature sensor, using *LabView*.

2.6 Properties of Ethanol

At $T = 25.5°\text{C}$:

- Index of refraction:
  $n = 1.3597$ at $\lambda = 589\text{nm}$
  $n = 1.3597$ at $\lambda = 656\text{nm}$
  (H_2O for comparison: $n = 1.332$)
- density $\rho = 0.785\text{g/cm}^3$
- for more properties, check literature.

2.7 Temperature dependency

Because of the strong temperature dependency of the measured SOS, it is important to record the temperature of the liquid during the experiments. Therefore it is required to implement a small *LabVIEW* program, which reads out the temperature in certain time intervals and simultaneously saves the corresponding diffraction pattern. It is possible, in addition, to read out the digital oscilloscope at the same time, for an indication of the used excitation frequency of the piezoelectric element. For the performance of the temperature dependent measurement of ethanol, cooled ethanol (8°C) will be provided by the lab course supervisor. To have an estimation for the temperature dependence, the SOS will be measured during the temperature increase from 8°C to room temperature (23°C). For an improved accuracy of the temperature coefficient, the measured temperature range can also be inverted by warming the ethanol up inside the glass cuvette , i.e. using warm water bath, and let it cool down until it reaches room temperature. The measured temperature range should be around room temperature +/- 15°C.

2.8 LabVIEW

For an overview and introduction into the software-package *LabVIEW* the following book [2] can be used. In addition many *LabVIEW* tutorials for basic programming can be found on the internet.

2.8.1 Integration of 'EyeToy PS 3 camera' in LabVIEW

For the integration of the 'PS3-camera' into *LabVIEW*, an additional software package NI-IMAQdx was added to the standard installation of *LabVIEW*. The additional functionalities and control buttons can be found in

LabVIEW-'Functions'-folder under 'Functions: Vision and Motion'. For a further explananation of how to control the camera in *LabVIEW*, see provided reference folder.

2.8.2 Integration of USB-temperature sensor in LabVIEW

To integrate the USB-temperature sensor Go!Temp, Go!Labview-Routines[1] have been installed, in addition to the standard installation of *LabVIEW*. These can be found under *LabVIEW*-'Functions'-Panel under 'User Libraries/GO!'. Example programs of how to integrate the USB-sensor can be found on the desktop of the lab computer.

2.9 Analysis of diffraction patterns using ImageJ

ImageJ is a 'public domain' image processing software (freeware).The software and documentation can be found and downloaded under http://rsbweb.nih.gov/ij/. To determine the distance between to diffraction maximums, the function 'Plot Profile' in the menu 'Analyze'[2] can be used. Note: The image analysis can also be performed using LabView or Matlab.

3 Excercises

3.1 Theoretical Preparation

Acquire knowledge over the following topics:

- Basic principle of the generation of ultrasonic waves (frequency range, piezoelectric elements, properties of ultrasound waves, Resonance frequency)
- Basic principle of propagation ultrasonic waves in a finite medium ('standing' waves, solution for wave equation for periodical excitation)
- Arrangement of components in experimental setup: Which purpose has each individual element in the setup (Figure 3)? What is the relation between the focal distance f_2 and the distance between the diffraction maximums?

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[1] http://www.vernier.com/go/gotemp.html

[2] http://rsbweb.nih.gov/ij/docs/menus/analyze.html

- Careful preparation of the SOS measurements in liquids using light diffraction by ultrasonic waves: Planing of the experiment and analysis, Estimation of influence of refraction at transition air-glass cuvette.
- Estimation of influence of different error sources (propagation of error)

3.2 Experiments

- Measurement of resonance frequencies of the piezoelectric element in water and determination of eigenfrequency with a high accuracy; comparison to theoretical value;
- Experimental determination of the speed of sound in ethanol using light diffraction by ultrasonic waves
- Create *LabVIEW* program for the experimental determination of the temperature coefficient of SOS in ethanol within a temperature range of around 8°C to 35°C. Apply determined temperature coefficient to compare measured SOS value with literature value (usually at 20°C).
- Discussion of influence of various error sources and estimation of relative and absolute measurement accuracy of the speed of sound

References

[1] L. Bergmann. *Der Ultraschall und seine Anwendung in Wissenschaft und Technik*. S. Hirzel Verlag Zürich, 5 edition, 1949.

[2] B. Mütterlein. *Handbuch für die Programmierung mit LabVIEW*. Elsevier, 2007. ISBN: 978-3-8274-1761-9.