Homework 22 (to be completed by October 1, 2019)

73) Verification of Parceval’s Equation

Consider the function \( f(x) = \exp(-a|x|) \) with \( a > 0 \).

a) Determine the Fourier transform \( \tilde{f}(k) \).

b) Use these functions to explicitly verify Parceval’s equation

\[
\int_{-\infty}^{\infty} dx \ |f(x)|^2 = 2\pi \int_{-\infty}^{\infty} dk \ |	ilde{f}(k)|^2.
\]

74) Harmonically Forced Harmonic Oscillator

Consider a weakly damped harmonic oscillator with a harmonic driving force described by the differential equation

\[
\ddot{x}(t) + \gamma \dot{x}(t) + \omega_0^2 x(t) = \cos(\omega t).
\]

Determine a particular solution following eq.(22.35) in the lecture notes.

a) Show that the integral in eq.(22.35) effectively extends from 0 to \( \infty \).

b) Perform the substitution \( t - t' = 2u/\gamma \) and solve the corresponding integral (e.g. using MAPLE). Show explicitly that the resulting particular solution indeed solves the differential equation.

c) Does this particular solution agree with the one of eq.(22.12)?

75) Strongly Damped Harmonic Oscillator

Closely follow the lecture notes describing the construction of a Green function and a particular solution for the weakly damped harmonic oscillator,
in order to extend this to the strongly damped oscillator.

a) What is the Green function in the strongly damped case?

b) Construct a particular solution for the harmonically forced strongly damped oscillator.