Homework 23
(to be completed by October 8, 2019)

76) Integrals with δ-Functions

Calculate the following integrals

a) \( \int_a^b dx \, \delta(x - 1) \),
b) \( \int dx \, \delta(ax + b) \),
c) \( \int dx \, \delta(a^2 - x^2) \exp(-bx) \),
d) \( \int dx \, \delta'(x + a) \exp(-bx) \),
e) \( \int dx \int dy \, \delta(x + y)\delta(x - y)f(x, y) \),
f) \( \int d^3x \, \delta(\vec{x} - \vec{y}) \exp[-(\vec{x} - \vec{a})^2/2b^2] \).

77) Differential Equations with δ-Functions

Solve the following differential equations

a) \( f'(x) = \delta(x - a) \), \quad f(\infty) = 0 ,
b) \( f''(x) = \delta(x - a) \exp(-bx) \), \quad f(\infty) = f'(\infty) = 0 ,
c) \( f''(x) = -f(x) + b \, \delta(x - a) \), \quad f(\infty) = f'(\infty) = 0 ,
d) Is there a symmetric solution of \( f''(x) = \delta(x) \)?
78) Green Function for a Differential Equation

Consider the differential equation $m\ddot{x}(t) = F(t)$ with $F(t) = 0$ for $t < 0$.

a) Construct a particular solution which obeys $x(t) = 0$ for $t < 0$.

b) Show that this solution can be obtained as

$$x(t) = \frac{1}{m} \int_{-\infty}^{\infty} dt' G(t-t') F(t').$$

What is the explicit form of the Green function $G(t)$?

c) Does this Green function reduce to the one of eq.(22.31) in the limits $\gamma \to 0$ and $\omega_0 \to 0$. 