Homework 30
(to be completed by December 13, 2019)

97) Integral along a Segment of \( \mathbb{C} \)

Evaluate the integral

\[
I = \int_{0}^{\infty} \frac{dx}{1 + x^N}, \quad N \in \mathbb{N}, \quad N \geq 2,
\]

by using the residue theorem. Integrate along the boundary of an appropriately chosen segment of the complex plane with an opening angle \( 2\pi/N \).

98) Integral along a Cut

Evaluate the integral

\[
I = \int_{-\infty}^{\infty} dx \frac{\log(x - i)}{(x + i)^2}
\]

by using the residue theorem. Close the contour in the upper half-plane and integrate on both sides of a cut that extends along the positive imaginary axis, starting at \( x = i \).

99) Green Function of the strongly damped Harmonic Oscillator

Consider the Green function of a damped harmonic oscillator that obeys

\[
\ddot{G}(t) + \gamma \dot{G}(t) + \omega_0^2 G(t) = \delta(t).
\]

a) Perform a Fourier transform and derive an equation for \( \tilde{G}(\omega) \).

b) Solve this equation in the strongly damped case \( \gamma^2/2 > \omega_0^2 \).

c) Perform a Fourier back transformation by applying the residue theorem.