Effective Field Theory

1st Exercise Sheet

1 Wilsonian EFT

1. Determine whether the operators

$$O_1 = \phi^6(x) \quad O_2 = \bar{\psi}(x)\psi(x)\,, \quad O_3 = \phi(x)\,\bar{\psi}(x)\psi(x)\,, \quad O_4 = \bar{\psi}(x)\psi(x)\,\bar{\psi}(x)\psi(x)$$

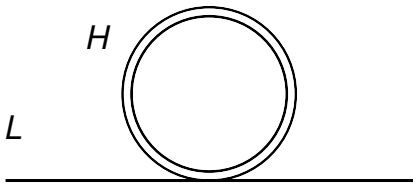
are relevant, marginal, or irrelevant. Derive first the mass dimensions of the scalar fields ϕ and spin-1/2 fields ψ by requiring that the free action be dimensionless (work in d dimensions). Consider then the above operators and study if/when their classification changes as a function of d.

2. Compute the surface Ω_d of the d-dimensional unit-sphere. To do so, it is useful to consider the integral

$$\left[\int_{-\infty}^{\infty} dk\, e^{-ak^2}\right]^d = \int_{-\infty}^{\infty} d^d k\, e^{-a\mathbf{k}^2} = \Omega_d \int_0^{\infty} dk\, k^{d-1}\, e^{-ak^2}\,.$$

After computing the integral over the length $k = |\mathbf{k}|$ one can solve for Ω_d. Check that you reproduce the known results for $d = 1, 2, 3$ and confirm the value for $d = 4$ used in the lecture. How does the surface behave when the dimension becomes large?

3. Consider scalar field theory in the Wilsonian framework, as we did in the lecture. Compute the one-loop self-energy correction from integrating out the high-energy part ϕ_H of the field:



This contribution modifies the mass term in the low-energy theory. By how much does the mass change if we integrate out the modes ϕ_H with momentum $b\Lambda < |k| < \Lambda$?