Effective Field Theory

2nd Exercise Sheet

2 Continuum EFT

Consider the following theory (in $d = 4$) with a heavy scalar ϕ_H and a light scalar ϕ_L:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_L\,\partial^\mu\phi_L - \frac{m^2}{2}\phi_L^2 + \frac{1}{2}\partial_\mu\phi_H\,\partial^\mu\phi_H - \frac{M^2}{2}\phi_H^2 \\ - \frac{\lambda_L}{4!}\phi_L^4 - \frac{\lambda_{HL}}{4}\phi_L^2\phi_H^2 - \frac{\lambda_H}{4!}\phi_H^4 - \frac{g}{2}\phi_H\phi_L^2.$$

At low energies $E \sim m \ll M$, the physics is described by an effective Lagrangian that depends only on the light field and has the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi_L\,\partial^\mu\phi_L - \frac{\tilde{m}^2}{2}\phi_L^2 - \frac{\tilde{\lambda}}{4!}\phi_L^4 - \frac{C_{2,4}}{2M^2}\phi_L\,\Box^2\,\phi_L - \frac{C_{4,2}}{4!M^2}\,\phi_L^2\,\Box\phi_L^2 - \frac{C_{6,0}}{6!M^2}\phi_L^6,$$

up to terms suppressed M^4.

1. Show that the above form of the Lagrangian $\mathcal{L}_{\text{eff}}$ is indeed the most general one up to dimension 6.

2. Perform a field redefinition in the effective theory

   $$\phi_L(x) \to \phi_L(x) + \frac{\alpha}{M^2}\,\Box\phi_L(x) + \frac{\beta}{M^2}\,\phi_L^3(x),$$

   dropping any $1/M^4$ terms. Show that a suitable choice of α and β eliminates the two operators $\phi_L\,\Box^2\,\phi_L$ and $\phi_L^2\,\Box\,\phi_L^2$ from $\mathcal{L}_{\text{eff}}$.

3. Draw the diagrams contributing to the two-point function up to one loop in the full and the effective theory. Use a double line to denote the heavy field and a single line for the light field.

4. Draw the diagrams contributing to the four-point function at one loop in the full and the effective theory. One representative of each topology is enough (additional diagrams, related by the exchange of external legs, need not be drawn).

3 QED in the vacuum sector

Consider QED in the vacuum sector ($N_{e^-} + N_{e^+} = 0$) at low energies $E_\gamma \ll m_e$. In this case, electrons and positrons only arise as virtual particles and can be integrated out, i.e., one can write down an effective Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ that contains only the photon field A_μ.

1. Write down the most general (gauge invariant!) Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ including operators up to dimension 4.

2. Write down the (minimal but complete) Lagrangian $\mathcal{L}_{\text{eff}}(A_\mu)$ including operators up to dimension 6.