Critical Phenomena - Exercise Set 4

10) Homogeneous functions

In this exercise we will derive some properties of homogeneous functions. Suppose that $f(x, y)$ is homogeneous, that is $f(\lambda^a x, \lambda^c y) = \lambda f(x, y)$.

a) Show that the derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are again homogeneous functions. What is their homogeneity relation?

b) Show that $f(x, y) = x^{2-\alpha} g\left(\frac{y}{x^\Delta}\right)$ for some function g.

c) What is the corresponding result if f is a homogeneous function of several variables?

11) Scaling relations

In this exercise we'll see how the scaling hypothesis leads to equalities between the critical exponents known as *scaling relations*. This fact has a deep connection with the renormalization group, which we'll see in the next weeks. Suppose that near the phase transition the free energy $f(t, h)$ (where $t \propto T - T_c$ and h is the magnetic field) is a homogeneous function, so that (see Exercise 10) $f(t, h) = |t|^{2-\alpha} g\left(\frac{h}{|t|^\Delta}\right)$.

1. Obtain the critical exponents β, δ, α, γ in terms of α and Δ. Was the name α chosen correctly?

2. Use the result from part a) to obtain the *scaling relations* between critical exponents.