

Series 1

I. Complex Gaussian integrals

1. Calculate the integral

$$\int (d\vec{z}^*)(d\vec{z}) e^{-(\vec{z}^*, A\vec{z})}, \quad (1)$$

where $\vec{z} = \frac{1}{\sqrt{2}}(\vec{x} + i\vec{y})$, $(d\vec{z}^*)(d\vec{z}) = (d\vec{x})(d\vec{y})$ and A is a real, symmetric N times N matrix.

II. Wick contractions

1. Calculate the expression

$$\langle x^{2n} \rangle = \frac{\int_{-\infty}^{+\infty} dx e^{-\frac{1}{2}ax^2} x^{2n}}{\int_{-\infty}^{+\infty} dx e^{-\frac{1}{2}ax^2}}. \quad (2)$$

2. Calculate the expression

$$\langle x_i x_j \rangle = \frac{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x} x_i x_j}{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x}}. \quad (3)$$

3. Calculate the expression

$$\langle x_i x_j x_k \rangle = \frac{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x} x_i x_j x_k}{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x}}. \quad (4)$$

4. Calculate the expression

$$\langle x_i x_j x_k x_l \rangle = \frac{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x} x_i x_j x_k x_l}{\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} dx_1 \dots dx_N e^{-\frac{1}{2}x \cdot A \cdot x}}. \quad (5)$$

5. What is the expression for a general number of insertions, $\langle x_i x_j \dots x_k x_l \rangle$?