Series 4

I. The potential energy in $d = 2 + 1$ dimensions

1. Calculate the analog of formula (1.69) in the notes in $2 + 1$ dimensions.

II. The massive vector meson

1. Starting from

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + A_\mu J^\mu, \tag{1}$$

arrive via partial integration at the action

$$S = \int \mathrm{d}^4 x \left(\frac{1}{2} A_\mu [(\partial^2 + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu] A_\nu + A_\mu J^\mu \right). \tag{2}$$

2. Find the propagator $D_{\nu\lambda}(k)$ from

$$[-(k^2 - m^2) g^{\mu\nu} + k^\mu k^\nu] D_{\nu\lambda}(k) = \delta^\mu_\lambda \tag{3}$$

by using the fact that it has to be symmetric in the indices and making an ansatz made of the possible symmetric pieces.

III. Continuous symmetry transformations

To put the formalism introduced in Sec. 2 to use, we apply it to the example of *scale transformations*, where the coordinates are rescaled by a positive number λ.

1. The finite transformation acts as

$$x \to \lambda x, \tag{4}$$

$$\Phi'(\lambda x) = \lambda^{-\Delta} \Phi(x), \tag{5}$$

where Δ is the so-called scaling dimension of the field. Find S'.

2. The infinitesimal scale transformation acts as

$$x \to x + \epsilon x \tag{6}$$

$$\mathcal{F}(\Phi) = (1 - \epsilon\Delta)\Phi. \tag{7}$$

Find the generator of the scale transformation.