Quantum Field Theory I — Exercise 2

Scattering Matrix & Feynman Rules — 17. 10. 2023

Consider the scalar Yukawa theory described by the following Lagrangian

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - M^2 \phi^\dagger \phi - \frac{1}{2} m^2 \varphi^2 - g\phi^\dagger \phi \varphi,$$

with $g \ll M, m$. This theory couples a complex scalar field ϕ, which we call nucleon field, to a real scalar φ which we interpret as a meson. We will investigate different scattering processes in this exercise sheet.

1. In this exercise we will look at the example of nucleon-nucleon scattering from the lecture $\phi\phi \to \phi\phi$ and calculate $\langle f|S|i\rangle$.

   (a) Write down the fields ϕ and φ in terms of creation and annihilation operators. Formulate the initial $|i\rangle$ and final states $|f\rangle$.

   (b) Argue why the first non-trivial contribution to the scattering matrix is given by

   $$\frac{(-ig)^2}{2} \int d^4x_1 d^4x_2 T\left(\phi^\dagger(x_1)\phi(x_1)\varphi(x_1)\phi^\dagger(x_2)\phi(x_2)\varphi(x_2)\right)$$

   (c) Apply Wick's theorem and simplify the expression.

   Hint: Before blindly considering all the possible contractions, think which of them actually contribute. Carefully looking at the intial and final states configurations might help.

2. We will now explore other scattering processes within the scalar Yukawa theory:

   (a) Consider $\phi\varphi \to \phi\varphi$. Follow the same steps as in 1.

   (b) Consider $\phi\phi^\dagger \to \varphi\varphi$. Follow the same steps as before.

   (c) Consider $\phi\phi^\dagger \to \phi\phi^\dagger$. Follow the same steps as before.

   (d) * What happens for pure meson scattering $\varphi\varphi \to \varphi\varphi$?

3. Re-interpret your findings for all the processes above as Feynman diagrams and

   (a) formulate Feynman rules;

   (b) check the consistency of your Feynman rules by re-deriving the previous results.