Quantum Field Theory I — Exercise 5

Yukawa Theory with Fermions — 20. 11. 2023

The interaction between a (Dirac) fermion ψ of mass m and a real scalar field ϕ of mass μ is governed by the Yukawa theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 + \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \lambda\phi\overline{\psi}\psi,$$

which is a generalisation of the toy model we considered in exercise 2. This theory couples fermions ψ, which we call nucleon field, to a real scalar ϕ which we interpret as a meson. We will investigate different scattering processes in this exercise sheet.

1. In this exercise we consider nucleon-nucleon scattering from the lecture $\psi\psi \to \psi\psi$.
   (a) Draw the corresponding diagrams.
   (b) Using the Feynman rules derived in the lecture, repeat the derivation of the amplitude.

2. We will now explore other scattering processes within the Yukawa theory:
   (a) Consider $\psi\overline{\psi} \to \phi\phi$. Write down the amplitude. Follow the same steps as in 1.
   (b) Consider $\psi\overline{\psi} \to \psi\overline{\psi}$. Write down the amplitude. Follow the same steps as before.
   (c) * What happens for pure meson scattering $\phi\phi \to \phi\phi$? You should find a so-called loop integral which you do not need to solve. Investigate the high-momentum limit, i.e. the ultraviolet behaviour of the integral. Is it finite?

3. Let us go back to the first exercise and consider again $\psi\psi \to \psi\psi$. Calculate the complex conjugate of the amplitude you obtained in exercise 1.