Quantum Field Theory I — Exercise 6

From MUonE to Scalar QED — 04. 12. 2023

1. In the theoretical exercises you looked at the process $e^+e^- \to \mu^+\mu^-$. However, from a phenomenological point of view, there is currently more interest in muon-electron scattering, i.e. $\mu^-e^- \to \mu^-e^-$. The calculation of muon-electron scattering is important in the context of the upcoming MUonE experiment at CERN and has been calculated up to next-to-next-to-leading-order accuracy.

   (a) Sketch the Feynman diagrams for $\mu^-e^- \to \mu^-e^-$. Calculate the matrix element and find the differential cross section in w.r.t the mandelstam variable t. Assume elastic scattering. Do not consider the electron or the muon as massless.

   (b) Take the massless limit of your result and verify by using crossing symmetry that you find the same result you already obtained in the theoretical exercises.

2. In this exercise we will investigate scalar QED, a theory which consists of a complex scalar field ϕ and a minimally coupled vector field A_μ. The Lagrangian reads

$$\mathcal{L} = (D_\mu\phi)(D^\mu\phi)^\dagger - m^2\phi^\dagger\phi - \frac{1}{4}\lambda\left(\phi^\dagger\phi\right)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

   with $F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$, and $D_\mu = \partial_\mu + ieA_\mu(x)$ is the covariant derivative coupling ϕ and A_μ.

   (a) Derive the Feynman rules for this theory.

   Hint: Try to expand the Lagrangian $\mathcal{L}$ and look at all its terms. Can you identify the ones for the propagators of the ϕ and A_μ fields? The other terms describe interactions. You should find three terms. In Figure 1 you find the interaction vertices with their associated Feynman rules. Can you identify which term in the Lagrangian corresponds to which Feynman rule?

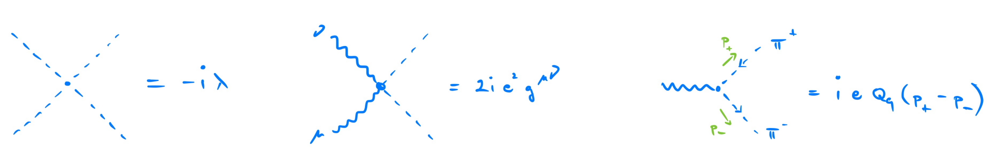

Figure 1: Feynman rules for the interactions in scalar QED.

   (b) Calculate the matrix element of the process $e^+e^- \to \pi^+\pi^-$ using the Feynman rules you derived in (a).