Quantum Field Theory I — Exercise 7

Massive vector meson — 18. 12. 2023

In this exercise sheet we will investigate a field theory describing a massive vector meson. To avoid any confusion we will denote the field of the massive vector meson with B_μ. The Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 B_\mu B^\mu,$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and M is the mass of the meson.

1. In this exercise we want to derive the propagator for the free theory, in other words determine the Green function using the functional approach.

   (a) Starting from the free action, show that the Green function $D_{\nu\rho}(x-y)$ in configuration space can be obtained as the solution to

   $$\left[\Box g^{\mu\nu} - \partial^\mu\partial^\nu + M^2 g^{\mu\nu}\right] D_{\nu\rho}(x-y) = i\delta^\mu_\rho \delta^{(4)}(x-y). \tag{1}$$

   (b) Solve (1) and find $D_{\nu\rho}(x-y)$.

   Hint: Transform to momentum space. To find the inverse matrix you are looking for, make an Ansatz for the general form of $\tilde{D}_{\nu\rho}$ in momentum space. Remember that by Lorentz invariance it is possible to write the most general form of any tensor structure with 2 components: $t_{\mu\nu} = Ag_{\mu\nu} + Bp_\mu p_\nu$, where A, B are coefficients which need to be determined.

   (c) Assume that the vector meson in massless, i.e. $M = 0$. Can you still solve (1)?

   Hint: Multiply (1) with ∂_μ.

2. Consider a massive vector meson minimally coupled to a Dirac particle, with coupling constant e according to the Lagrangian above. Compute, to lowest nontrivial order in perturbation theory, the amplitude for elastic fermion-antifermion scattering and explicitly verify that the contribution of the term in the vector meson propagator proportional to $\frac{k^\mu k^\nu}{M^2}$ vanishes. (It is not necessary to do spin sums or compute cross sections, or even to simplify the amplitude to demonstrate the desired result—but if you are curious, nothing can stop you from calculating the cross section too!).

3. (a) In the theory of the previous problem, compute the amplitude for elastic vector-spinor scattering, again to lowest nontrivial order. Verify that if the vector meson spin vector, ε_μ, is aligned with its four momentum k_μ , for either the incoming or the outgoing meson, the amplitude vanishes, even when the meson in question is off-shell (but the other particles are on-shell). Of course, what you are verifying is that it has a vanishing divergence between initial and final states defined by the on-shell particles.

(b) The same problem, but this time with a scalar particle rather than a Dirac particle, i.e. in scalar QED. You can find the Feynman rules for scalar QED in Figure 1.

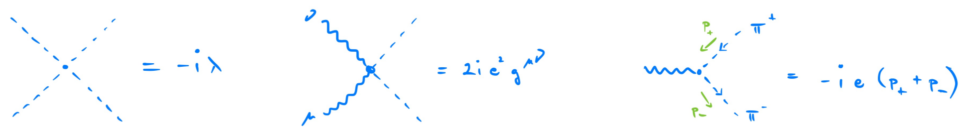

Figure 1: Feynman rules for the interactions in scalar QED.