8.1 One consequence of our new formulation of scattering theory is that it doesn't matter much what local field you assign to a particle: any field that has a properly normalized vacuum to one-particle matrix element will give the right S matrix element. (See the discussion following (14.35).)

Consider the theory of a free scalar field,

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\mu^2\phi^2 \qquad \text{(P8.1)}$$

Let us define a new field, A, by

$$\phi = A + \tfrac{1}{2}gA^2 \qquad \text{(P8.2)}$$

In terms of A, the Lagrangian becomes

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu A)^2(1+gA)^2 - \tfrac{1}{2}\mu^2(A + \tfrac{1}{2}gA^2)^2 \qquad \text{(P8.3)}$$

If you had been presented with this Lagrangian, and didn't know its origin, you would probably think it described a highly nontrivial theory, with complicated non-zero scattering amplitudes. Of course, you do know its origin, and thus you know that it must predict vanishing scattering. Verify this by actually summing up all the graphs that contribute to meson–meson elastic scattering in the A-field formulation, to lowest nontrivial order in g, i.e., g^2, and showing that the sum vanishes.

Comments:

(1) Our general theory does not tell us that the A field Green's functions are the same as the ϕ field Green's functions, so the amplitudes may not vanish if the external momenta are not on the mass shell.

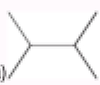
(2) To the order in which we are working, we can completely ignore renormalization counterterms.

(3) This is a theory with derivative interactions. As discussed in class, this leads to potential problems: the interaction Lagrangian is not the same as minus the interaction Hamiltonian, and we can't pull time derivatives through the time-ordering symbol in Dyson's formula. Much later in this course, we shall study such theories using the methods of functional integration, and discover that (to this order in perturbation theory) these problems cancel. Take this on trust here; use the naive Feynman rules as explained in class. (That is to say, treat the theory as if the interaction Hamiltonian were minus the interaction Lagrangian, and as if every derivative ∂_μ became a factor of $-ip_\mu$ for an incoming momentum, and ip_μ for an outgoing one.)

(4) If you have an interaction proportional to A^4, there are 4! different ways of choosing which fields annihilate and which create which mesons. If you don't keep proper track of these (and similar) factors, you'll never get the right answer.

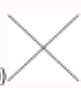
(5) A graph of order g^2 may contain either one vertex proportional to g^2 or two vertices each proportional to g.

(6) To get you started, here are the graphs you'll have to study (with various momenta on the external lines):


(a)

Each vertex here derived from

$$\mathscr{L}_I = \cdots + gA(\partial_\mu A)^2 - \tfrac{1}{2}g\mu^2A^3 + \cdots$$


(b)

Vertex here derived from

$$\mathscr{L}_I = \cdots + \tfrac{1}{2}g^2A^2(\partial_\mu A)^2 - \tfrac{1}{8}\mu^2g^2A^4 + \cdots$$