Quantum Field Theory II FS24

Series 7 - Loop calculations in spin-$\frac{1}{2}$ theories 17.04.2024

In this series, we will calculate the **triangle diagram** we calculated in the part 2 of series 4, but in the spin-$\frac{1}{2}$ theories. In particular, we consider the Lagrangian of the Model 3 discussed in Coleman's chapter 23.

Consider the triangle diagram for an incoming nucleon of mass m and momentum p, an outgoing nucleon of mass m and momentum p' and an incoming meson with mass μ and momentum $q = p' - p$. In the following, consider the on-shell relations $p^2 = p'^2 = m^2$, wherever needed.

1. Prove that the expression for this diagram can be written as:

$$-g^3 \bar{u}(p')\gamma_5 \; I \; u(p), \tag{1}$$

with

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{(\not{p}' + \not{k} + m)\;\gamma_5\;(\not{p} + \not{k} + m)\;\gamma_5}{D_1\;D_2\;D_3}, \tag{2}$$

$$D_1 = (p' + k)^2 - m^2, \qquad D_2 = (p + k)^2 - m^2, \qquad D_3 = k^2 - \mu^2. \tag{3}$$

2. Decompose the integral I in terms of the following integrals

$$C_{\{0,\mu,\mu\nu\}} = \int \frac{d^4k}{(2\pi)^4} \frac{\{1, k_\mu, k_\mu k_\nu\}}{D_1 D_2 D_3}. \tag{4}$$

Hint: Remember that $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ *and* $\{\gamma^\mu, \gamma_5\} = 0$.

3. Show that the term proportional to C_0 is zero.

Hint: Consider the Dirac equation for the spinors.

4. Simplify the term proportional to C_μ, by considering the decomposition $C_\mu = p_\mu C_1 + p'_\mu C_2$.

5. Prove that the final result of the contribution of the triangle diagram is

$$-g^3 \bar{u}(p')\gamma_5 u(p) G(q^2) \tag{5}$$

and determine the function $G(q^2)$ in terms of $C_{0,1,2}$ as well as B_0, which is defined as

$$B_0 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{D_1 D_2}. \tag{6}$$

Notice that you have already calculated the integrals B_0 and C_0 in the previous series of this course and their results are known.

6. Work out explicit expressions (as loop integrals) for C_1 and C_2.

Hint: Use the Feynman parametrization for the integral of C_μ and remember how you takled a similar integral in the part 2 of series 4. You might also find the following formula useful

$$\int \frac{d^4k'}{(2\pi)^4} \frac{1}{(k'^2 + a)^3} = \frac{i}{32\pi^2 a}. \tag{7}$$

7. *Optional:* Provide a general decomposition of the loop integral $C_{\mu\nu}$ in terms of Lorentz tensors of second rank built out of the two momenta p_μ and p'_μ and the metric tensor $g_{\mu\nu}$ multiplying scalar loop functions.