Series 1

I. Dimensional analysis

1. Work out the engineering dimension of the free fermion in 4d obeying

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi. \tag{1}$$

2. Work out the engineering dimension of the real scalar field a in 3d with kinetic term $\frac{1}{2}\partial_\mu a\partial^\mu a$.

3. Based on the engineering dimension of a, what kind of potential term can we add to the Lagrangian if we use only dimensionless couplings?

4. Consider a complex scalar field of the form $\phi = ae^{i\chi}$ in 3d. Its Lagrangian contains (among others) the kinetic term for a and the potential term determined above. What must be the dimension of χ?

5. What other terms made of a and χ can be added to the Lagrangian (again without using dimensionful couplings and up tp two derivatives in the fields)?

II. Transformations from the inversion

1. Perform explicitly an inversion

$$x^\mu \to \frac{x^\mu}{x^2} \tag{2}$$

followed by a translation by a^μ followed by another inversion. What is the resulting transformation?

2. Perform explicitly an inversion followed by a dilatation/scale transformation followed by another inversion. What is the resulting transformation?